thm_2Enumpair_2Enpair (TMUfV8zERnDvkJVN6JZPRbkBWVvBc4B1nQc)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 3 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Enumpair\_2Einvtri0 : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Einvtri0 \in (((ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enumpair_2Einvtri *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Epair\_2ESND\ ty$

Let $c\_2Enumpair\_2Etri : \iota$ be given. Assume the following.

$$c\_2Enumpair\_2Etri \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Definition 6 *We define* c_2Enumpair_2Enfst *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E$

Definition 7 *We define* c_2Enumpair_2Ensnd *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E$

Definition 8 *We define* c_2Enumpair_2Enpair *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enumpair\_2Enfst}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V0x)\ V1y)) = V0x))) \tag{12}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enumpair\_2Ensnd}\ (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V0x)\ V1y)) = V1y))) \tag{13}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\exists V1x \in ty\_2Enum\_2Enum.(\exists V2y \in ty\_2Enum\_2Enum.(V0n = (ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ V1x)\ V2y))))) \tag{14}$$

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ \mathsf{c\_2Enumpair\_2Enpair}\ (ap\ \mathsf{c\_2Enumpair\_2Enfst}\ V0n))\ (ap\ \mathsf{c\_2Enumpair\_2Ensnd}\ V0n)) = V0n))$$