

thm\_2Enumposrep\_2ELOG\_l2n  
(TMSf8pT1QgMCYR3oVZPqj4vdjQx8Ey9vzjg)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{2}$$

Let  $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p\ (ap\ P\ x)) \mathbf{then}$  (the  $(\lambda x. x \in A \wedge p\ x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ P)))$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2. V0x))\ (\lambda V1x \in 2. V1x)))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a})\ P))))$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2. V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. V2t))))$

**Definition 9** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (ap\ (c\_2Emin\_2E\_3D\_3D\_3E\ V1t1)\ V2t2))))$

Let  $c\_2Elogroot\_2ELOG : \iota$  be given. Assume the following.

$$c\_2Elogroot\_2ELOG \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

**Definition 10** We define  $c\_2E\_marker\_2E\_abbrev$  to be  $\lambda V0x \in 2.V0x$ .

Let  $c\_2E\_arithmic\_2E\_even : \iota$  be given. Assume the following.

$$c\_2E\_arithmic\_2E\_even \in (2^{ty\_2E\_enum\_2E\_enum}) \quad (5)$$

Let  $c\_2E\_arithmic\_2E\_odd : \iota$  be given. Assume the following.

$$c\_2E\_arithmic\_2E\_odd \in (2^{ty\_2E\_enum\_2E\_enum}) \quad (6)$$

**Definition 11** We define  $c\_2E\_bool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2E\_min\_2E\_3D\_3D\_3E V0t) c\_2E\_bool\_2E\_7E))$

Let  $c\_2E\_enum\_2E\_erep\_num : \iota$  be given. Assume the following.

$$c\_2E\_enum\_2E\_erep\_num \in (\omega^{ty\_2E\_enum\_2E\_enum}) \quad (7)$$

Let  $c\_2E\_enum\_2E\_suc\_rep : \iota$  be given. Assume the following.

$$c\_2E\_enum\_2E\_suc\_rep \in (\omega^{\omega}) \quad (8)$$

Let  $c\_2E\_enum\_2E\_abs\_num : \iota$  be given. Assume the following.

$$c\_2E\_enum\_2E\_abs\_num \in (ty\_2E\_enum\_2E\_enum^{\omega}) \quad (9)$$

**Definition 12** We define  $c\_2E\_enum\_2E\_suc$  to be  $\lambda V0m \in ty\_2E\_enum\_2E\_enum.(ap c\_2E\_enum\_2E\_abs\_num m)$

**Definition 13** We define  $c\_2E\_prim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2E\_enum\_2E\_enum.\lambda V1n \in ty\_2E\_enum\_2E\_enum$

**Definition 14** We define  $c\_2E\_arithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2E\_enum\_2E\_enum.\lambda V1n \in ty\_2E\_enum\_2E\_enum$

**Definition 15** We define  $c\_2E\_bool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2E\_bool\_2E\_21 2) (\lambda V2t \in 2))))$

**Definition 16** We define  $c\_2E\_arithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2E\_enum\_2E\_enum.\lambda V1n \in ty\_2E\_enum\_2E\_enum$

Let  $c\_2E\_arithmic\_2E\_exp : \iota$  be given. Assume the following.

$$c\_2E\_arithmic\_2E\_exp \in ((ty\_2E\_enum\_2E\_enum^{ty\_2E\_enum\_2E\_enum})^{ty\_2E\_enum\_2E\_enum}) \quad (10)$$

Let  $c\_2E\_arithmic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2E\_arithmic\_2E\_2D \in ((ty\_2E\_enum\_2E\_enum^{ty\_2E\_enum\_2E\_enum})^{ty\_2E\_enum\_2E\_enum}) \quad (11)$$

**Definition 17** We define  $c\_2E\_arithmic\_2E\_enumerable$  to be  $\lambda V0x \in ty\_2E\_enum\_2E\_enum.V0x$ .

**Definition 18** We define  $c\_2E\_enumerable\_2E\_iisuc$  to be  $\lambda V0n \in ty\_2E\_enum\_2E\_enum.(ap c\_2E\_enum\_2E\_suc (ap$

**Definition 19** We define  $c\_2E\_enumerable\_2E\_iz$  to be  $\lambda V0x \in ty\_2E\_enum\_2E\_enum.V0x$ .

Let  $c\_2E\_enum\_2E\_zero\_rep : \iota$  be given. Assume the following.

$$c\_2E\_enum\_2E\_zero\_rep \in \omega \quad (12)$$



Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (20)$$

Let  $c\_2Elist\_2ELAST : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELAST\ A\_27a \in (A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)} \quad (21)$$

**Definition 28** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap\ V1f\ V0x)))$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ c\_2Enum\_2E0) = V0m)) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap\ c\_2Enum\_2ESUC\ V1n)))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ (ap\ c\_2Enum\_2ESUC\ V0m))\ V1n)))))) \quad (25)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ c\_2Enum\_2E0)\ V0n))) \quad (26)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\neg (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1n)\ V0m)))))) \quad (27)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ c\_2Enum\_2E0) = c\_2Enum\_2E0)) \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap \\
& (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A \\
& V0m) V1n))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2B V0m) \\
& V2p) = (ap (ap c\_2Earithmetic\_2E\_2B V1n) V2p)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))) \Rightarrow (p ( \\
& ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = c\_2Enum\_2E0) \Leftrightarrow ((V0m = \\
& c\_2Enum\_2E0) \vee (V1n = c\_2Enum\_2E0))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (\neg (V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC \\
& V1n)) V0m))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0n))) \quad (35)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V0m)) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0m)) \wedge (V1n = c\_2Enum\_2E0)) \wedge ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n)) \Leftrightarrow ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V1n)) \wedge (V0m = c\_2Enum\_2E0)))))) \quad (36)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1k)\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V1k)))) \quad (37)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow (((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = c\_2Enum\_2E0) \Leftrightarrow (\exists V2d \in ty\_2Enum\_2Enum.(V1k = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V2d)\ V0n)))))) \quad (38)$$

Assume the following.

$$True \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (46)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg (p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p \ V0t)))))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg (p \ V0A)) \vee (\neg (p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg (p \ V0A)) \wedge (\neg (p \ V1B)))))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg (p \ V0A)) \vee (p \ V1B)))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (55)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool\_2ECOND} A_{.27a}) \\ V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool\_2ECOND} A_{.27a}) V1Q) V3x_{.27}) \\ V5y_{.27})))))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ A_{.27a}.((ap (ap (ap (c_{.2Ebool\_2ECOND} A_{.27a}) c_{.2Ebool\_2ET} V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ (ap (ap (c_{.2Ebool\_2ECOND} A_{.27a}) c_{.2Ebool\_2EF} V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}). \\ (\forall V2x \in A_{.27c}.((ap (ap (ap (c_{.2Ecombin\_2Eo} A_{.27c} A_{.27b} A_{.27a}) \\ V0f) V1g) V2x) = (ap V0f (ap V1g V2x)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ nonempty A_{.27c} \Rightarrow (\forall V0f \in ((A_{.27c}^{A_{.27b}})^{A_{.27a}}).(\forall V1x \in \\ A_{.27b}.(\forall V2y \in A_{.27a}.((ap (ap (ap (c_{.2Ecombin\_2EC} A_{.27a} A_{.27b} \\ A_{.27c}) V0f) V1x) V2y) = (ap (ap V0f V2y) V1x)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (((ap (c_{.2Elist\_2ELENGTH} A_{.27a}) \\ (c_{.2Elist\_2ENIL} A_{.27a})) = c_{.2Enum\_2E0}) \wedge (\forall V0h \in A_{.27a}.( \\ \forall V1t \in (ty\_2Elist\_2Elist A_{.27a}).((ap (c_{.2Elist\_2ELENGTH} \\ A_{.27a}) (ap (ap (c_{.2Elist\_2ECONS} A_{.27a}) V0h) V1t)) = (ap c_{.2Enum\_2ESUC} \\ (ap (c_{.2Elist\_2ELENGTH} A_{.27a}) V1t)))))) \end{aligned} \quad (61)$$



Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0P \in (2^{A\_27a}).((p\ (ap \\
& (ap\ (c\_2Elist\_2EVERY\ A\_27a)\ V0P)\ (c\_2Elist\_2ENIL\ A\_27a))) \Leftrightarrow True)) \wedge \\
& (\forall V1P \in (2^{A\_27a}).(\forall V2h \in A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist \\
& A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EVERY\ A\_27a)\ V1P)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V2h)\ V3t))) \Leftrightarrow ((p\ (ap\ V1P\ V2h)) \wedge (p\ (ap\ (ap\ (c\_2Elist\_2EVERY \\
& A\_27a)\ V1P)\ V3t)))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\
& (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\
& A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ ( \\
& c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\
& A\_27a).(p\ (ap\ V0P\ V3l))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0l \in (ty\_2Elist\_2Elist \\
& A\_27a).(V0l = (c\_2Elist\_2ENIL\ A\_27a)) \vee (\exists V1h \in A\_27a.( \\
& \exists V2t \in (ty\_2Elist\_2Elist\ A\_27a).(V0l = (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V1h)\ V2t))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a1 \in (ty\_2Elist\_2Elist \\
& A\_27a).(\forall V1a0 \in A\_27a.(\neg((c\_2Elist\_2ENIL\ A\_27a) = (ap\ ( \\
& ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V1a0)\ V0a1))))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1l \in \\
& (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Elist\_2EVERY\ A\_27a) \\
& V0P)\ V1l)) \Leftrightarrow (\forall V2e \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& V2e)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V1l))) \Rightarrow (p\ (ap\ V0P\ V2e))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (ap \\
& (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a) \\
& (c\_2Elist\_2ENIL\ A\_27a)))) \Leftrightarrow False)) \wedge (\forall V1x \in A\_27a.(\forall V2h \in \\
& A\_27a.(\forall V3t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& A\_27a)\ V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& A\_27a)\ V2h)\ V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& V1x)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET\ A\_27a)\ V3t)))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a. (\forall V1t \in \\ & (ty\_2Elist\_2Elist\ A\_27a). ((ap\ (c\_2Elist\_2ELAST\ A\_27a)\ (ap\ (ap \\ & (c\_2Elist\_2ECONS\ A\_27a)\ V0h)\ V1t)) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\ & A\_27a)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (ty\_2Elist\_2Elist\ A\_27a))\ V1t)\ ( \\ & c\_2Elist\_2ENIL\ A\_27a)))\ V0h)\ (ap\ (c\_2Elist\_2ELAST\ A\_27a)\ V1t)))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & (\forall V0a \in ty\_2Enum\_2Enum. (\forall V1b \in ty\_2Enum\_2Enum. ( \\ & ((p\ (ap\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Earithmetic\_2ENUMERAL \\ & (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))\ V0a)) \wedge ( \\ & p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V1b))) \Rightarrow (((ap\ (ap\ c\_2Elogroot\_2ELOG \\ & V0a)\ V1b) = c\_2Enum\_2E0) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1b)\ V0a)))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\ & \forall V2y \in ty\_2Enum\_2Enum. (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\ & (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \\ & V0b)) \wedge ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V2y)) \wedge (p\ ( \\ & ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1x)\ V0b)))))) \Rightarrow ((ap\ (ap\ c\_2Elogroot\_2ELOG \\ & V0b)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A \\ & V0b)\ V2y))\ V1x)) = (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Elogroot\_2ELOG\ V0b) \\ & V2y)))))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \quad (71)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = V0n) \wedge (((ap\ c\_2Enumeral\_2EiZ\ ( \\
& ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ ( \\
& ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ ( \\
& ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enumeral\_2EiZ\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ V0n)) \wedge (((ap \\
& c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge ( \\
& ((ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ V1m)))) \wedge \\
& (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ c\_2Earithmetic\_2EZERO) \\
& V0n)) = (ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC \\
& (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0n)\ c\_2Earithmetic\_2EZERO)) = ( \\
& ap\ c\_2Enumeral\_2EiiSUC\ V0n)) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ ( \\
& ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m))) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enum\_2ESUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT1\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m)))) \wedge (((ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& (ap\ c\_2Earithmetic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmetic\_2EBIT2\ V1m))) = \\
& (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiiSUC\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B \\
& V0n)\ V1m))))))))))))))))))))))))))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D c\_2Earithmic\_2EZERO) V0n)) \Leftrightarrow \\
& \quad True) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& \quad V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2EBIT2 V0n)) c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge \\
& \quad (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& \quad V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT1 \\
& \quad V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& \quad V0n)) (ap c\_2Earithmic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2EBIT2 \\
& \quad V0n)) (ap c\_2Earithmic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad V0n) V1m))))))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0b \in ty\_2Enum\_2Enum. ((ap (ap c\_2Enumposrep\_2El2n V0b) \\
& \quad (c\_2Elist\_2ENIL ty\_2Enum\_2Enum)) = c\_2Enum\_2E0)) \wedge (\forall V1b \in \\
& \quad ty\_2Enum\_2Enum. (\forall V2h \in ty\_2Enum\_2Enum. (\forall V3t \in ( \\
& \quad ty\_2Elist\_2Elist ty\_2Enum\_2Enum). ((ap (ap c\_2Enumposrep\_2El2n \\
& \quad V1b) (ap (ap (c\_2Elist\_2ECONS ty\_2Enum\_2Enum) V2h) V3t)) = (ap ( \\
& \quad ap c\_2Earithmic\_2E\_2B (ap (ap c\_2Earithmic\_2EMOD V2h) V1b)) \\
& \quad (ap (ap c\_2Earithmic\_2E\_2A V1b) (ap (ap c\_2Enumposrep\_2El2n \\
& \quad V1b) V3t))))))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0b \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad c\_2Enum\_2E0) V0b)) \Rightarrow (\forall V1l \in (ty\_2Elist\_2Elist ty\_2Enum\_2Enum). \\
& \quad (((ap (ap c\_2Enumposrep\_2El2n V0b) V1l) = c\_2Enum\_2E0) \Leftrightarrow (p (ap ( \\
& \quad ap (c\_2Elist\_2EVERY ty\_2Enum\_2Enum) (ap (ap (c\_2Ecombin\_2Eo \\
& \quad ty\_2Enum\_2Enum 2 ty\_2Enum\_2Enum) (ap (c\_2Emin\_2E\_3D ty\_2Enum\_2Enum) \\
& \quad c\_2Enum\_2E0)) (ap (ap (c\_2Ecombin\_2EC ty\_2Enum\_2Enum ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum) c\_2Earithmic\_2EMOD) V0b))) V1l))))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad ((ap c\_2Enum\_2ESUC V0m) = (ap c\_2Enum\_2ESUC V1n)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (\forall V0m \in \\
& \quad ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE (ap c\_2Enum\_2ESUC V0m)) = \\
& \quad V0m)))
\end{aligned} \tag{78}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0n) c\_2Enum\_2E0)))) \quad (79)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0e \in A\_27a. (\forall V1l \in \\ & (ty\_2Elist\_2Elist A\_27a). (p (ap (ap (c\_2Ebool\_2EIN A\_27a) (ap \\ & (c\_2Elist\_2ELAST A\_27a) (ap (ap (c\_2Elist\_2ECONS A\_27a) V0e) V1l))) \\ & (ap (c\_2Elist\_2ELIST\_TO\_SET A\_27a) (ap (ap (c\_2Elist\_2ECONS \\ & A\_27a) V0e) V1l)))))) \end{aligned} \quad (80)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (83)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (84)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (85)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\ & (p V2r)) \vee (\neg(p V1q)))) \wedge ((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{88}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (( \\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{89}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{90}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{92}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V0p))) \tag{93}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{94}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p)) \tag{95}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0b \in ty\_2Enum\_2Enum. ((p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \\
& (ap \ c\_2Earithmetic\_2ENUMERAL \ (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO))) \\
& V0b)) \Rightarrow (\forall V1l \in (ty\_2Elist\_2Elist \ ty\_2Enum\_2Enum). (((\neg \\
& (V1l = (c\_2Elist\_2ENIL \ ty\_2Enum\_2Enum))) \wedge ((p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \\
& c\_2Enum\_2E0) \ (ap \ (c\_2Elist\_2ELAST \ ty\_2Enum\_2Enum) \ V1l))) \wedge (p \\
& (ap \ (ap \ (c\_2Elist\_2EVERY \ ty\_2Enum\_2Enum) \ (ap \ c\_2Earithmetic\_2E\_3E \\
& V0b) \ V1l)))) \Rightarrow ((ap \ (ap \ c\_2Elogroot\_2ELOG \ V0b) \ (ap \ (ap \ c\_2Enumposrep\_2El2n \\
& V0b) \ V1l)) = (ap \ c\_2Eprim\_rec\_2EPRE \ (ap \ (c\_2Elist\_2ELENGTH \ ty\_2Enum\_2Enum) \\
& V1l))))))
\end{aligned}$$