thm_2Eone_2Eone__axiom (TMaDfmuUYbtcr-Venb16nEvYGRQkK5eSqFEg)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 8 *We define* c_2Ebool_2ETYPE__DEFINITION *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0P \in (2^{A\_27a}).(\lambda V$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ETYPE\_\_DEFINITION}\ A\_27a\ A\_27b)\ V0P)\ V1rep)) \Leftrightarrow ((\forall V2x\_27 \in A\_27b.(\forall V3x\_27\_27 \in A\_27b.(((ap\ V1rep\ V2x\_27) = (ap\ V1rep\ V3x\_27\_27)) \Rightarrow (V2x\_27 = V3x\_27\_27)))) \wedge (\forall V4x \in A\_27a. ((p\ (ap\ V0P\ V4x)) \Leftrightarrow (\exists V5x\_27 \in A\_27b.(V4x = (ap\ V1rep\ V5x\_27))))))))) \tag{3}$$

Assume the following.

$$(\exists V0rep \in (2^{ty\_2Eone\_2Eone}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ETYPE\_\_DEFINITION}\ 2\ ty\_2Eone\_2Eone)\ (\lambda V1b \in 2.V1b))\ V0rep))) \tag{4}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (ty\_2Eone\_2Eone^{A\_27a}).(\forall V1g \in (ty\_2Eone\_2Eone^{A\_27a}).(V0f = V1g)))$$