

# thm\_2Eoption\_2EIS\_SOME\_MAP (TMHdtqtTUBL4SbCLJn29EQ1vFXgt616s7En)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_EF$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 9** We define  $c\_2Ebool\_2E\_5C\_2E\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (1)$$

Let  $c\_2Eoption\_2EOPTION\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_MAP A\_27a A\_27b \in (((ty\_2Eoption\_2Eoption A\_27b)^{(ty\_2Eoption\_2Eoption A\_27a)})^{(A\_27b^{A\_27a})}) \quad (2)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \quad (3)$$

**Definition 10** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone$

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \quad (4)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (5)$$

**Definition 12** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (6)$$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) ($

**Definition 14** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS$

**Definition 15** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_$

Let  $c\_2Eoption\_2EIS\_SOME : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2EIS\_SOME A\_27a \in (2^{(ty\_2Eoption\_2Eoption A\_27a)}) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption A\_27a).((V0opt = (c\_2Eoption\_2ENONE A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap (c\_2Eoption\_2ESOME A\_27a) V1x)))))) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V0f)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V1x)) = (ap\ (c\_2Eoption\_2ESOME \\ A\_27b)\ (ap\ V0f\ V1x)))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V2f)\ (c\_2Eoption\_2ENONE\ A\_27a)) = (c\_2Eoption\_2ENONE \\ A\_27b)))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (c\_2Eoption\_2EIS\_SOME \\ A\_27a)\ (ap\ (c\_2Eoption\_2ESOME\ A\_27a)\ V0x))) \Leftrightarrow True)) \wedge ((p\ (ap\ (c\_2Eoption\_2EIS\_SOME \\ A\_27a)\ (c\_2Eoption\_2ENONE\ A\_27a))) \Leftrightarrow False)) \end{aligned} \quad (13)$$

**Theorem 1**

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in (ty\_2Eoption\_2Eoption \\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A\_27b)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_MAP \\ A\_27a\ A\_27b)\ V0f)\ V1x))) \Leftrightarrow (p\ (ap\ (c\_2Eoption\_2EIS\_SOME\ A\_27a) \\ V1x)))))) \end{aligned}$$