# thm_2Eoption_2EOPTION__APPLY__MAP2 (TMKyC1oDuC7MXwaz49bwYP4Vt9xcEzMvwUa)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 6** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

**Definition 7** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 8** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 9** *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

**Definition 10** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

**Definition 11** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

**Definition 12** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $c\_2Eoption\_2Eoption\_\_ABS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a\in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

**Definition 13** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ (c$

Let $c\_2Eoption\_2ETHE:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2ETHE\ A\_27a\in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{6}$$

**Definition 14** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e\in A\_27a.(ap\ (c\_2Esum\_2EABS$

**Definition 15** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Eoption\_2EIS\_\_SOME:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a\in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{7}$$

**Definition 16** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a:\iota.(\lambda V0t\in 2.(\lambda V1t1\in A\_27a.(\lambda V2t2\in A\_27a.($

**Definition 17** *We define* c_2Eoption_2EOPTION__MAP2 *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in ((A$

Let $c\_2Eoption\_2EOPTION\_\_MAP:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b\in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{8}$$

Let $c\_2Eoption\_2EOPTION\_\_APPLY:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Eoption\_2EOPTION\_\_APPLY\ A\_27a\ A\_27b\in (((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ A\_27b)})^{(ty\_2Eoption\_2Eoption\ (A\_27a^{A\_27b} \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$(\forall V0t\in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x))))) \wedge (\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27a^{A\_27c})^{A\_27b}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27c.((((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27c)\ V2y)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ (ap\ (ap\ V0f\ V1x)\ V2y))) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27c)\ V2y)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1o1 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2o2 \in (ty\_2Eoption\_2Eoption\ A\_27b).(((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27c\ A\_27a\ A\_27b)\ V0f)\ V1o1)\ V2o2) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c)) \Leftrightarrow ((V1o1 = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (V2o2 = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27b).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_APPLY\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ (A\_27a^{A\_27b})))\ V0x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \wedge (\forall V1f \in (A\_27a^{A\_27b}).(\forall V2x \in (ty\_2Eoption\_2Eoption\ A\_27b).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_APPLY\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (A\_27a^{A\_27b}))\ V1f))\ V2x) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27b\ A\_27a)\ V1f)\ V2x))))) \tag{19}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2y \in (ty\_2Eoption\_2Eoption\ A\_27b).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_APPLY\ A\_27c\ A\_27b)\ (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ (A\_27c^{A\_27b}))\ V0f)\ V1x))\ V2y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27c\ A\_27a\ A\_27b)\ V0f)\ V1x)\ V2y)))))$$