

thm_2Eoption_2EOPTION_IGNORE_BIND_EQUALS_OPTION
(TMJTm-
cjyZW6ypY3FmNvxYXAoCFZsNz8wJRb)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 7 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 8 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (3)$$

Definition 11 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (5)$$

Definition 12 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 13 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone.\ V0x))$

Definition 14 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 15 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ (c_2Eone_2Eone\ A_27a))$

Definition 16 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let $c_2Eoption_2EOPTION_BIND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eoption_2EOPTION_BIND\ A_27a\ A_27b \in (((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Eoption_2Eoption\ A_27a)^{A_27b}})^{(ty_2Eoption_2Eoption\ A_27b)}) \quad (6)$$

Definition 17 We define $c_2Eoption_2EOPTION_IGNORE_BIND$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0m1 \in (ty_2Eoption_2EOPTION_BIND\ A_27a\ A_27b)$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (9)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\exists V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (10)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{11}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{12}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{13}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True))
\end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1a \in \\
& A_27a.((\exists V2x \in A_27a.((V2x = V1a) \wedge (p \ (ap \ V0P \ V2x)))) \Leftrightarrow (p \ (\\
& ap \ V0P \ V1a))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0opt \in (ty_2Eoption_2Eoption \\
& A_27a).((V0opt = (c_2Eoption_2ENONE \ A_27a)) \vee (\exists V1x \in A_27a. \\
& (V0opt = (ap \ (c_2Eoption_2ESOME \ A_27a) \ V1x))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.(((ap \ (c_2Eoption_2ESOME \ A_27a) \ V0x) = (ap \ (c_2Eoption_2ESOME \\
& A_27a) \ V1y)) \Leftrightarrow (V0x = V1y))))
\end{aligned} \tag{19}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg((c_2Eoption_2ENONE\ A_27a) = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0m \in (ty_2Eoption_2Eoption\ A_27a). (\\ \forall V1v \in A_27c. (((ap\ (ap\ (c_2Eoption_2EOPTION_IGNORE_BIND \\ A_27a\ A_27b)\ (c_2Eoption_2ENONE\ A_27b))\ V0m) = (c_2Eoption_2ENONE \\ A_27a)) \wedge ((ap\ (ap\ (c_2Eoption_2EOPTION_IGNORE_BIND\ A_27a\ A_27c) \\ (ap\ (c_2Eoption_2ESOME\ A_27c)\ V1v))\ V0m) = V0m)))))) \end{aligned} \quad (21)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0m1 \in (ty_2Eoption_2Eoption\ A_27a). (\forall V1m2 \in (ty_2Eoption_2Eoption \\ A_27b). (\forall V2y \in A_27b. (((ap\ (ap\ (c_2Eoption_2EOPTION_IGNORE_BIND \\ A_27b\ A_27a)\ V0m1)\ V1m2) = (c_2Eoption_2ENONE\ A_27b)) \Leftrightarrow ((V0m1 = \\ (c_2Eoption_2ENONE\ A_27a)) \vee (V1m2 = (c_2Eoption_2ENONE\ A_27b)))))) \wedge \\ (((ap\ (ap\ (c_2Eoption_2EOPTION_IGNORE_BIND\ A_27b\ A_27a)\ V0m1) \\ V1m2) = (ap\ (c_2Eoption_2ESOME\ A_27b)\ V2y)) \Leftrightarrow (\exists V3x \in A_27a. \\ ((V0m1 = (ap\ (c_2Eoption_2ESOME\ A_27a)\ V3x)) \wedge (V1m2 = (ap\ (c_2Eoption_2ESOME \\ A_27b)\ V2y)))))))))) \end{aligned}$$