

# thm\_2Eoption\_2EOPTION\_IGNORE\_BIND\_thm (TMSV<sub>y</sub>V1qBNJxMD6Zv8GvRbAACgsXnnwuiSq)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \tag{3}$$

**Definition 6** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

**Definition 7** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (V0x))$

**Definition 8** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 9** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone.V0x))$

**Definition 10** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 11** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21))$

**Definition 12** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS A\_27a A\_27b) (V0e))$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (V0x))$

**Definition 14** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let  $c\_2Eoption\_2EOPTION\_BIND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_BIND \\ & A\_27a A\_27b \in (((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{A\_27b}})(ty\_2Eoption\_2Eoption A\_27a A\_27b)) \end{aligned} \quad (6)$$

**Definition 15** We define  $c\_2Eoption\_2EOPTION\_IGNORE\_BIND$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0m1 \in (ty\_2Eoption\_2Eoption A\_27a A\_27b).V0m1$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap (ap (c\_2Ecombin\_2EK A\_27a A\_27b) V0x) V1y) = V0x))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & (\forall V0f \in ((ty\_2Eoption\_2Eoption A\_27a)^{A\_27b}).((ap (ap (c\_2Eoption\_2EOPTION\_BIND A\_27a A\_27b) (c\_2Eoption\_2ENONE A\_27b)) \\ & V0f) = (c\_2Eoption\_2ENONE A\_27a))) \wedge (\forall V1x \in A\_27b.(\forall V2f \in \\ & ((ty\_2Eoption\_2Eoption A\_27a)^{A\_27b}).((ap (ap (c\_2Eoption\_2EOPTION\_BIND A\_27a A\_27b) (ap (c\_2Eoption\_2ESOME A\_27b) V1x)) V2f) = (ap V2f V1x)))))) \end{aligned} \quad (10)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow \forall A\_27c. \\ & nonempty A\_27c \Rightarrow (\forall V0m \in (ty\_2Eoption\_2Eoption A\_27a). \\ & (\forall V1v \in A\_27c.(((ap (ap (c\_2Eoption\_2EOPTION\_IGNORE\_BIND A\_27a A\_27b) (c\_2Eoption\_2ENONE A\_27b)) V0m) = (c\_2Eoption\_2ENONE \\ & A\_27a)) \wedge ((ap (ap (c\_2Eoption\_2EOPTION\_IGNORE\_BIND A\_27a A\_27c) \\ & (ap (c\_2Eoption\_2ESOME A\_27c) V1v)) V0m) = V0m)))))) \end{aligned}$$