thm_2Eoption_2EOPTION__MAP2__cong (TMY-FuTJyMq4477iDXHHvdyWRLqTjK6o7Moh)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 7 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A$

Definition 8 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Definition 9 *We define* c_2Eone_2Eone *to be* $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \tag{3}$$

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

Definition 13 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{6}$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{7}$$

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Eoption_2EOPTION__MAP2 *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A$

Assume the following.

$$True \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{12}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow(((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\vee(\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y))\Leftrightarrow(V0x = V1y)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0f \in ((A\_27a^{A\_27c})^{A\_27b}).(\forall V1x \in A\_27b.(\forall V2y \in A\_27c.(((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27c)\ V2y)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ (ap\ (ap\ V0f\ V1x)\ V2y)))\wedge(((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\wedge(((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27c)\ V2y)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27c)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))))))) \tag{16}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0x1 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V1x2 \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2y1 \in (ty\_2Eoption\_2Eoption\ A\_27b).(\forall V3y2 \in (ty\_2Eoption\_2Eoption\ A\_27b).(\forall V4f1 \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V5f2 \in ((A\_27c^{A\_27b})^{A\_27a}).(((V0x1 = V1x2)\wedge((V2y1 = V3y2)\wedge(\forall V6x \in A\_27a.(\forall V7y \in A\_27b.(((V1x2 = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V6x))\wedge(V3y2 = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V7y)))\Rightarrow((ap\ (ap\ V4f1\ V6x)\ V7y) = (ap\ (ap\ V5f2\ V6x)\ V7y)))))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27c\ A\_27a\ A\_27b)\ V4f1)\ V0x1)\ V2y1) = (ap\ (ap\ (ap\ (\mathsf{c\_2Eoption\_2EOPTION\_\_MAP2}\ A\_27c\ A\_27a\ A\_27b)\ V5f2)\ V1x2)\ V3y2)))))))))$$