# thm_2Eoption_2EOPTION__MAP__CASE (TMHdztxzKHFte2ZGkBZQNTpuRsq49J4Az7B)

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**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

**Definition 4** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

**Definition 5** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 6** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

**Definition 7** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 8** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{1}$$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \tag{2}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{3}$$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{4}$$

Let $c\_2Eoption\_2EIS\_\_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{5}$$

**Definition 9** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{6}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{7}$$

**Definition 10** *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

**Definition 11** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

**Definition 12** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{8}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{9}$$

**Definition 13** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{10}$$

**Definition 14** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{11}$$

**Definition 15** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

**Definition 16** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}). (\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a. (V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{15}$$

Assume the following.

$$
\begin{gathered}
\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
\forall V0e \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2e \in ( \\
ty\_2Eoption\_2Eoption\ A\_27a).((\forall V3x \in A\_27a.(\forall V4y \in \\
A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V4y)) \Leftrightarrow (V3x = V4y)))) \wedge ((\forall V5x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE \\
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V5x)) = V5x)) \wedge ((\forall V6x \in \\
A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V6x)))) \wedge ((\forall V7x \in A\_27a.(\neg((ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V7x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \wedge ((\forall V8x \in A\_27a. \\
((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ V8x))) \Leftrightarrow True)) \wedge (((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a) \\
(\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False) \wedge ((\forall V9x \in (ty\_2Eoption\_2Eoption \\
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V9x)) \Leftrightarrow (V9x = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a)))) \wedge ((\forall V10x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg \\
(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V10x))) \Leftrightarrow (V10x = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a)))) \wedge ((\forall V11x \in (ty\_2Eoption\_2Eoption\ A\_27a).((p \\
(ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V11x)) \Rightarrow ((ap\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V11x)) = V11x))) \wedge ((\forall V12x \in \\
(ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V12x)\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a))\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = V12x)) \wedge ((\forall V13x \in ( \\
ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V13x)\ V13x)\ (\mathsf{c\_2Eoption\_2ESOME} \\
A\_27a)) = V13x)) \wedge ((\forall V14x \in (ty\_2Eoption\_2Eoption\ A\_27a). \\
((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V14x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ A\_27b)\ V14x)\ V0e)\ V1f) = V0e))) \wedge ((\forall V15x \in (ty\_2Eoption\_2Eoption \\
A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V15x)) \Rightarrow ((ap\ (ap \\
(ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ V15x)\ V0e)\ V1f) = ( \\
ap\ V1f\ (ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ V15x))))) \wedge ((\forall V16x \in \\
(ty\_2Eoption\_2Eoption\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME \\
A\_27a)\ V16x)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ ( \\
ty\_2Eoption\_2Eoption\ A\_27a))\ V16x)\ V2e)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = \\
V16x))) \wedge ((\forall V17v \in A\_27b.(\forall V18f \in (A\_27b^{A\_27a}).( \\
(ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27a))\ V17v)\ V18f) = V17v))) \wedge ((\forall V19x \in A\_27a.(\forall V20v \in \\
A\_27b.(\forall V21f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE \\
A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V19x))\ V20v)\ V21f) = \\
(ap\ V21f\ V19x))))) \wedge ((\forall V22f \in (A\_27b^{A\_27a}).(\forall V23x \in \\
A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V22f)\ ( \\
ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V23x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b) \\
(ap\ V22f\ V23x))))) \wedge ((\forall V24f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP \\
A\_27a\ A\_27b)\ V24f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE} \\
A\_27b))) \wedge (((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE} \\
(ty\_2Eoption\_2Eoption\ A\_27a))) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge \\
(\forall V25x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN \\
A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Eoption\_2Eoption\ A\_27a)) \\
V25x)) = V25x))))))))))))))))))))))))
\end{gathered}
\tag{16}
$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27b))\ V1x)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27b)\ A\_27b)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b))\ V0f)))))$$