

thm_2Eoption_2EOPTION__MAP__EQ__NONE__both__ways
(TM-
FQe1qHM1SC8HAFqg1YvWMNVdJezbjVpH2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone.V0x))$

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_2Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c$
Let $c_2Eoption_2EOPTION_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Eoption_2EOPTION_MAP A_27a A_27b \in (((ty_2Eoption_2Eoption A_27b)^{(ty_2Eoption_2Eoption A_27a)})^{(A_27b^{A_27a})}) \quad (6)$$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (9)$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in (ty_2Eoption_2Eoption \\ & A_27a). (((ap (ap (c_2Eoption_2EOPTION_MAP A_27a A_27b) V0f) \\ & V1x) = (c_2Eoption_2ENONE A_27b)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE A_27a)))))) \end{aligned} \quad (11)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27a^{A_27b}). (\forall V1x \in (ty_2Eoption_2Eoption \\ & A_27b). (((ap (ap (c_2Eoption_2EOPTION_MAP A_27b A_27a) V0f) \\ & V1x) = (c_2Eoption_2ENONE A_27a)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE A_27b))) \wedge \\ & (((c_2Eoption_2ENONE A_27a) = (ap (ap (c_2Eoption_2EOPTION_MAP \\ & A_27b A_27a) V0f) V1x)) \Leftrightarrow (V1x = (c_2Eoption_2ENONE A_27b)))))) \end{aligned}$$