

# thm\_2Eoption\_2Eoption\_\_Axiom (TMGMT- GcZdL9HjEyT1x6RVyNuvGe5EBJshg)

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**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E21$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $(ap (c\_2Ebool\_2E21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E3D\_3D\_3E V0t) c\_2Ebool\_2E21))$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{3}$$

Let  $c\_2Eoption\_2Eoption\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_REP\ A\_27a \in ((ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{4}$$

**Definition 7** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (5)$$

**Definition 8** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS\ A\_27a \in \\ ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \end{aligned} \quad (6)$$

**Definition 9** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0x)$

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A)\ P)$  of type  $\iota \Rightarrow \iota$ .

**Definition 11** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone.V0x))$

**Definition 12** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS\_sum\ A\_27a\ A\_27b)\ V0e)$

**Definition 13** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ V0e)$

**Definition 14** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a)\ V0P)))$

Assume the following.

$$True \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ (p\ V0t)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Eoption\_2Eoption \\
& A\_27a).(ap\ (c\_2Eoption\_2Eoption\_ABS\ A\_27a)\ (ap\ (c\_2Eoption\_2Eoption\_REP \\
& A\_27a)\ V0a)) = V0a)) \wedge (\forall V1r \in (ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone). \\
& ((p\ (ap\ (\lambda V2x \in (ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone).c\_2Ebool\_2ET) \\
& V1r)) \Leftrightarrow ((ap\ (c\_2Eoption\_2Eoption\_REP\ A\_27a)\ (ap\ (c\_2Eoption\_2Eoption\_ABS \\
& A\_27a)\ V1r)) = V1r)))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}).(\forall V1g \in (A\_27c^{A\_27b}). \\
& (\exists V2h \in (A\_27c^{(ty\_2Esum\_2Esum\ A\_27a\ A\_27b)}).((\forall V3x \in \\
& A\_27a.((ap\ V2h\ (ap\ (c\_2Esum\_2EINL\ A\_27a\ A\_27b)\ V3x)) = (ap\ V0f\ V3x))) \wedge \\
& (\forall V4y \in A\_27b.((ap\ V2h\ (ap\ (c\_2Esum\_2EINR\ A\_27a\ A\_27b)\ V4y)) = \\
& (ap\ V1g\ V4y)))))) \\
& \hspace{15em} (13)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0e \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).(\exists V2fn \in \\
& (A\_27b^{(ty\_2Eoption\_2Eoption\ A\_27a)}).(((ap\ V2fn\ (c\_2Eoption\_2ENONE \\
& A\_27a)) = V0e) \wedge (\forall V3x \in A\_27a.((ap\ V2fn\ (ap\ (c\_2Eoption\_2ESOME \\
& A\_27a)\ V3x)) = (ap\ V1f\ V3x)))))) \\
& \hspace{15em}
\end{aligned}$$