thm_2Eoption_2Eoption__CLAUSES (TMWTHz-ZKEU8P7yZmCRyswp96Yx9ta9zbNc6)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (1)$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \quad (2)$$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (3)$$

Let $c\_2Eoption\_2EOPTION\_\_JOIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Eoption\_2Eoption\ (ty\_2Eoption\_2Eoption\ A\_27a))}) \quad (4)$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Eoption\_2EIS\_\_NONE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_NONE\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (5)$$

Let $c\_2Eoption\_2EIS\_\_SOME : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2EIS\_\_SOME\ A\_27a \in (2^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (6)$$

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (7)$$

Definition 11 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (8)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (9)$$

Definition 12 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (10)$$

Definition 13 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 14 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 15 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (11)$$

Assume the following.

$$True \quad (12)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg ((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg ((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ (ap\ V0f\ V1x)))))\wedge(\forall V2f \in (A\_27b^{A\_27a}).((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V2f)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x))) \Leftrightarrow True))\wedge((p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow False)) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x))) \Leftrightarrow False))\wedge((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))) \Leftrightarrow True)) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)) = V0x)) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eoption\_2Eoption\ A\_27a))) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\wedge(\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (c\_2Eoption\_2EOPTION\_\_JOIN\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0x)) = V0x))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((p\ (ap\ (c\_2Eoption\_2EIS\_\_NONE\ A\_27a)\ V0x)) \Leftrightarrow (V0x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{27}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((\neg(p\ (ap\ (c\_2Eoption\_2EIS\_\_SOME\ A\_27a)\ V0x))) \Leftrightarrow (V0x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0x)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = V0x)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eoption\_2Eoption\ A\_27a).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0x)\ V0x)\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)) = V0x)) \tag{30}$$

Theorem 1

$$\begin{gathered}
\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
\forall V0e \in A_27b.(\forall V1f \in (A_27b^{A_27a}).(\forall V2e \in (\\
ty_2Eoption_2Eoption\ A_27a).((\forall V3x \in A_27a.(\forall V4y \in \\
A_27a.(((ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V3x) = (ap\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)\ V4y)) \Leftrightarrow (V3x = V4y)))) \wedge ((\forall V5x \in A_27a.((ap\ (c_2Eoption_2ETHE \\
A_27a)\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V5x)) = V5x)) \wedge ((\forall V6x \in \\
A_27a.(\neg((\mathsf{c_2Eoption_2ENONE}\ A_27a) = (ap\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)\ V6x)))) \wedge ((\forall V7x \in A_27a.(\neg((ap\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)\ V7x) = (\mathsf{c_2Eoption_2ENONE}\ A_27a)))) \wedge ((\forall V8x \in A_27a. \\
((p\ (ap\ (c_2Eoption_2EIS__SOME\ A_27a)\ (ap\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)\ V8x))) \Leftrightarrow True)) \wedge (((p\ (ap\ (c_2Eoption_2EIS__SOME\ A_27a) \\
(\mathsf{c_2Eoption_2ENONE}\ A_27a))) \Leftrightarrow False) \wedge ((\forall V9x \in (ty_2Eoption_2Eoption \\
A_27a).((p\ (ap\ (c_2Eoption_2EIS__NONE\ A_27a)\ V9x)) \Leftrightarrow (V9x = (\mathsf{c_2Eoption_2ENONE} \\
A_27a)))) \wedge ((\forall V10x \in (ty_2Eoption_2Eoption\ A_27a).((\neg \\
(p\ (ap\ (c_2Eoption_2EIS__SOME\ A_27a)\ V10x))) \Leftrightarrow (V10x = (\mathsf{c_2Eoption_2ENONE} \\
A_27a)))) \wedge ((\forall V11x \in (ty_2Eoption_2Eoption\ A_27a).((p \\
(ap\ (c_2Eoption_2EIS__SOME\ A_27a)\ V11x)) \Rightarrow ((ap\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)\ (ap\ (c_2Eoption_2ETHE\ A_27a)\ V11x)) = V11x))) \wedge ((\forall V12x \in \\
(ty_2Eoption_2Eoption\ A_27a).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE \\
A_27a\ (ty_2Eoption_2Eoption\ A_27a))\ V12x)\ (\mathsf{c_2Eoption_2ENONE} \\
A_27a))\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)) = V12x)) \wedge ((\forall V13x \in (\\
ty_2Eoption_2Eoption\ A_27a).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE \\
A_27a\ (ty_2Eoption_2Eoption\ A_27a))\ V13x)\ V13x)\ (\mathsf{c_2Eoption_2ESOME} \\
A_27a)) = V13x)) \wedge ((\forall V14x \in (ty_2Eoption_2Eoption\ A_27a). \\
((p\ (ap\ (c_2Eoption_2EIS__NONE\ A_27a)\ V14x)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE \\
A_27a\ A_27b)\ V14x)\ V0e)\ V1f) = V0e))) \wedge ((\forall V15x \in (ty_2Eoption_2Eoption \\
A_27a).((p\ (ap\ (c_2Eoption_2EIS__SOME\ A_27a)\ V15x)) \Rightarrow ((ap\ (ap \\
(ap\ (c_2Eoption_2Eoption__CASE\ A_27a\ A_27b)\ V15x)\ V0e)\ V1f) = (\\
ap\ V1f\ (ap\ (c_2Eoption_2ETHE\ A_27a)\ V15x))))) \wedge ((\forall V16x \in \\
(ty_2Eoption_2Eoption\ A_27a).((p\ (ap\ (c_2Eoption_2EIS__SOME \\
A_27a)\ V16x)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE\ A_27a\ (\\
ty_2Eoption_2Eoption\ A_27a))\ V16x)\ V2e)\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)) = \\
V16x))) \wedge ((\forall V17v \in A_27b.(\forall V18f \in (A_27b^{A_27a}).(\\
(ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE\ A_27a\ A_27b)\ (\mathsf{c_2Eoption_2ENONE} \\
A_27a))\ V17v)\ V18f) = V17v))) \wedge ((\forall V19x \in A_27a.(\forall V20v \in \\
A_27b.(\forall V21f \in (A_27b^{A_27a}).((ap\ (ap\ (ap\ (c_2Eoption_2Eoption__CASE \\
A_27a\ A_27b)\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V19x))\ V20v)\ V21f) = \\
(ap\ V21f\ V19x))))) \wedge ((\forall V22f \in (A_27b^{A_27a}).(\forall V23x \in \\
A_27a.((ap\ (ap\ (c_2Eoption_2EOPTION__MAP\ A_27a\ A_27b)\ V22f)\ (\\
ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27a)\ V23x)) = (ap\ (\mathsf{c_2Eoption_2ESOME}\ A_27b) \\
(ap\ V22f\ V23x))))) \wedge ((\forall V24f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Eoption_2EOPTION__MAP \\
A_27a\ A_27b)\ V24f)\ (\mathsf{c_2Eoption_2ENONE}\ A_27a)) = (\mathsf{c_2Eoption_2ENONE} \\
A_27b))) \wedge (((ap\ (c_2Eoption_2EOPTION__JOIN\ A_27a)\ (\mathsf{c_2Eoption_2ENONE} \\
(ty_2Eoption_2Eoption\ A_27a))) = (\mathsf{c_2Eoption_2ENONE}\ A_27a)) \wedge \\
(\forall V25x \in (ty_2Eoption_2Eoption\ A_27a).((ap\ (c_2Eoption_2EOPTION__JOIN \\
A_27a)\ (ap\ (\mathsf{c_2Eoption_2ESOME}\ (ty_2Eoption_2Eoption\ A_27a)) \\
V25x)) = V25x))))))))))))))))))))))))
\end{gathered}$$