

thm_2Eordinal_2EIFF_ZERO_lt
(TMHjQd513Dh5tdHsVXYRXC4zeTNHWE m9Zsw)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \tag{4}$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) (ty_2Ewellorder_2Ewellorder\ A_27a)) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E_2C))$

Definition 9 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

Definition 10 We define `c_2Epair_2EUNCURRY` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

Definition 11 We define `c_2Eset_relation_2Estrict` to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 12 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) p)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40))))$

Definition 14 We define `c_2Eset_relation_2Erange` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 15 We define `c_2Eset_relation_2Edomain` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 16 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

Definition 17 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EUNION))$

Definition 18 We define `c_2Ewellorder_2EelsOf` to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 19 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Eorderiso)$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))}})$$

(11)

Definition 20 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota. \lambda V0r \in (ty_2Ewellorder_2Ewellorder)$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS\ A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)})$$

(12)

Definition 21 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 22 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 23 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}})$$

(13)

Definition 24 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 25 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 26 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 27 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 28 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 29 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

Definition 30 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap\ (ap\ c_2Earithmetic$

Definition 31 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Eordinal_2EfromNat\ A.27a \in ((ty_2Eordinal_2Eordinal\ A.27a)^{ty_2Enum_2Enum}) \quad (19)$$

Definition 32 We define $c_2Eordinal_2Eoleast$ to be $\lambda A.27a : \iota. \lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}). (ap$

Definition 33 We define $c_2Eordinal_2EordSUC$ to be $\lambda A.27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A.27a)$

Definition 34 We define $c_2Ecombin_2EK$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0x \in A.27a. (\lambda V1y \in A.27b. V0x))$

Definition 35 We define $c_2Ecombin_2ES$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda A.27c : \iota. (\lambda V0f \in ((A.27c^{A.27b})^{A.27a}))$

Definition 36 We define $c_2Ecombin_2EI$ to be $\lambda A.27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A.27a\ (A.27a^{A.27a}))\ A.27a))$

Definition 37 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda A.27c : \iota. \lambda A.27d : \iota. \lambda V0f$

Definition 38 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V0R1 \in ((2^{A.27a})^{A.27b})$

Definition 39 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. \lambda V0R \in ((2^{A.27a})^{A.27b})$

Definition 40 We define $c_2Ecombin_2EW$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (\lambda V0f \in ((A.27b^{A.27a})^{A.27a}). (\lambda V1x$

Definition 41 We define $c_2Equotient_2Erespects$ to be $\lambda A.27a : \iota. \lambda A.27b : \iota. (c_2Ecombin_2EW\ A.27a\ A.27b)$

Definition 42 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A.27a : \iota. (\lambda V0p \in (2^{A.27a}). (\lambda V1m \in (2^{A.27a}).$

Definition 43 We define $c_2Equotient_2EEQUIV$ to be $\lambda A.27a : \iota. \lambda V0E \in ((2^{A.27a})^{A.27a}). (ap\ (c_2Ebool_2E$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow False) \Leftrightarrow \neg (p \ V0t)))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg (p \ V0t)))))) \quad (29)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (32)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((\text{ap } (c_2Ecombin_2EI A_{27a}) V0x) = V0x)) \quad (33)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (p (\text{ap } (\text{ap } (\text{ap } (c_2Equotient_2EQUOTIENT (ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_{27a})) (ty_2Eordinal_2Eordinal A_{27a})) (c_2Ewellorder_2Eorderiso (ty_2Esum_2Esum ty_2Enum_2Enum A_{27a})) (ty_2Esum_2Esum ty_2Enum_2Enum A_{27a})) (c_2Eordinal_2Eordinal_ABS A_{27a})) (c_2Eordinal_2Eordinal_REP A_{27a}))) \quad (34)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_{27a}). (\neg (p (\text{ap } (\text{ap } (c_2Eordinal_2Eordlt A_{27a}) V0a) (\text{ap } (c_2Eordinal_2EfromNat A_{27a}) c_2Enum_2E0)))))) \quad (35)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_{27a}). (\forall V1b \in (ty_2Eordinal_2Eordinal A_{27a}). ((p (\text{ap } (\text{ap } (c_2Eordinal_2Eordlt A_{27a}) V0a) (\text{ap } (c_2Eordinal_2EordSUC A_{27a}) V1b))) \Leftrightarrow ((p (\text{ap } (\text{ap } (c_2Eordinal_2Eordlt A_{27a}) V0a) V1b)) \vee (V0a = V1b)))))) \quad (36)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c_2Eordinal_2EordSUC A_{27a}) (\text{ap } (c_2Eordinal_2EfromNat A_{27a}) c_2Enum_2E0)) = (\text{ap } (c_2Eordinal_2EfromNat A_{27a}) (\text{ap } c_2Earithmetic_2ENUMERAL (\text{ap } c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (c_2Ecombin_2EI\ A_27a))) \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c)\ \\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2)))))))))) \quad (39) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\ A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \quad (40) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\ A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\ V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\ (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \quad (41) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2ERespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P))))))
\end{aligned} \tag{47}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee \neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p))))))
\end{aligned} \tag{57}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{58}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q)))) \tag{59}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V0p)))) \tag{60}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V1q)))) \tag{61}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\
& A_27a).(p \ (ap \ (ap \ (c_2Ewellorder_2Eorderiso \ A_27a \ A_27a) \ V0w) \\
& V0w)))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\ & \quad (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27b\ A_27a)\ V1w2)\ V0w1)))))) \end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). \\ & \quad (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2w3 \in \\ & \quad (ty_2Ewellorder_2Ewellorder\ A_27c).(((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27b\ A_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27c)\ V0w1)\ V2w3)))))) \end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\ & \quad (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \vee ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \vee (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\ & \quad A_27b\ A_27a)\ V1w2)\ V0w1)))))) \end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0x0 \in (\\ & \quad ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1y0 \in (ty_2Ewellorder_2Ewellorder \\ & \quad A_27b).(\forall V2a0 \in (ty_2Ewellorder_2Ewellorder\ A_27c).(\\ & \quad \forall V3b0 \in (ty_2Ewellorder_2Ewellorder\ A_27d).(((p\ (ap\ (ap \\ & \quad (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap \\ & \quad (c_2Ewellorder_2Eorderiso\ A_27c\ A_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap \\ & \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap \\ & \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27b\ A_27d)\ V1y0)\ V3b0)))))) \end{aligned} \tag{67}$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ & \quad A_27a).(((\neg(V0x = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))) \Leftrightarrow \\ & \quad (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\ & \quad A_27a)\ c_2Enum_2E0))\ V0x))) \wedge ((\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ & \quad A_27a)\ V0x)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ (ap\ c_2Earithmic_2ENUMERAL \\ & \quad (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO)))))) \Leftrightarrow (p\ (\\ & \quad ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\ & \quad A_27a)\ c_2Enum_2E0))\ V0x)))) \end{aligned}$$