

thm\_2Eordinal\_2Ecsup\_\_thm  
(TMKYC6TFzn535Wwp7K7xt6ZJ3HjwzsKzjj8)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A$ .if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 4** We define  $c\_2Epred\_set\_2EUNIV$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2E\_ET)$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).ap V1f V0x))$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2$

**Definition 9** We define  $c\_2Epred\_set\_2EINJ$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A\_27b})$

**Definition 10** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 11** We define  $c\_2Epred\_set\_2Ecountable$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).ap (c\_2Ebool\_2E\_3F$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \tag{2}$$

Let  $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \quad (3)$$

Let  $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \quad (4)$$

Let  $c\_2Eordinal\_2Eordinal\_REP\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \quad (5)$$

**Definition 12** We define  $c\_2Eordinal\_2Eordinal\_REP$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Ewellorder\_2Ewellorder\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \quad (7)$$

**Definition 13** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 14** We define  $c\_2Ebool\_2E7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E21\ 2))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (8)$$

**Definition 15** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b.(ap\ (c\_2Ebool\_2E7E\ (ap\ (c\_2Ebool\_2E21\ 2)\ V0x)\ V1y))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (9)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (10)$$

**Definition 16** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c)^{A\_27a})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (11)$$

**Definition 17** We define  $c\_2Eset\_relation\_2Estrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

**Definition 18** We define  $c\_2Ewellorder\_2Eiseg$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 19** We define  $c\_2Eset\_relation\_2Errestrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

Let  $c\_2Ewellorder\_2Ewellorder\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_ABS \\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \end{aligned} \quad (12)$$

**Definition 20** We define  $c\_2Ewellorder\_2Ewobound$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 21** We define  $c\_2Eset\_relation\_2Erangle$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 22** We define  $c\_2Eset\_relation\_2Edomain$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 23** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2. (c\_2Ebool\_2E\_21\ 2)\ t))))$

**Definition 24** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2E\_21\ 2)\ t))$

**Definition 25** We define  $c\_2Ewellorder\_2EelsOf$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 26** We define  $c\_2Ewellorder\_2Eorderiso$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 27** We define  $c\_2Ewellorder\_2Eorderlt$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 28** We define  $c\_2Eordinal\_2Eordlt$  to be  $\lambda A\_27a : \iota. \lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

**Definition 29** We define  $c\_2Eordinal\_2Epreds$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

**Definition 30** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b)^{A\_27a}. \lambda V1s \in (2^{A\_27a})$

**Definition 31** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap\ (c\_2Epred\_set\_2E\_21\ 2)\ P))$

**Definition 32** We define  $c\_2Eordinal\_2Eoleast$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})$

**Definition 33** We define  $c\_2Eordinal\_2Esup$  to be  $\lambda A\_27a : \iota. \lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})$

**Definition 34** We define  $c\_2Ecardinal\_2Ecardleq$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0s1 \in (2^{A\_27a}). \lambda V1s2 \in (2^{A\_27a})$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (16)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. (((\forall V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow (\forall V3x \in A.27a. ((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. ((p (ap V1P V3x)) \vee (p V0Q)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. ((p (ap V1Q V3x)))))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. ((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{.27} \in 2. (\forall V2y \in 2. (\forall V3y_{.27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow \forall A_{.27b}. \text{nonempty } A_{.27b} \Rightarrow \forall A_{.27c}. \\ \text{nonempty } A_{.27c} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). (\forall V1t \in (2^{A_{.27b}}). \\ (\forall V2u \in (2^{A_{.27c}}). (((p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } \\ A_{.27a} A_{.27b}) V0s) V1t)) \wedge (p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } A_{.27b} \\ A_{.27c}) V1t) V2u))) \Rightarrow (p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } A_{.27a} A_{.27c}) \\ V0s) V2u)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{A_{.27a}}). ((p (ap (c_{.2} \text{Epred}_{.set} \text{Ecountable } A_{.27a}) V0s)) \Leftrightarrow (p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } A_{.27a} \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum}) V0s) (c_{.2} \text{Epred}_{.set} \text{EUNIV } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum})))))) \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0s \in (2^{(\text{ty}_{.2} \text{Eordinal}_{.2} \text{Eordinal } A_{.27a})}). \\ ((p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } (\text{ty}_{.2} \text{Eordinal}_{.2} \text{Eordinal } \\ A_{.27a}) (\text{ty}_{.2} \text{Esum}_{.2} \text{Esum } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum } A_{.27a})) V0s) (c_{.2} \text{Epred}_{.set} \text{EUNIV } \\ (\text{ty}_{.2} \text{Esum}_{.2} \text{Esum } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum } A_{.27a})))) \Rightarrow (\forall V1a \in (\text{ty}_{.2} \text{Eordinal}_{.2} \text{Eordinal } \\ A_{.27a}). ((p (ap (ap (c_{.2} \text{Eordinal}_{.2} \text{Eordlt } A_{.27a}) V1a) (ap (c_{.2} \text{Eordinal}_{.2} \text{Esup } \\ A_{.27a}) V0s))) \Leftrightarrow (\exists V2b \in (\text{ty}_{.2} \text{Eordinal}_{.2} \text{Eordinal } A_{.27a}). \\ ((p (ap (ap (c_{.2} \text{Ebool}_{.2} \text{EIN } (\text{ty}_{.2} \text{Eordinal}_{.2} \text{Eordinal } A_{.27a}) V2b) \\ V0s)) \wedge (p (ap (ap (c_{.2} \text{Eordinal}_{.2} \text{Eordlt } A_{.27a}) V1a) V2b)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (p (ap (ap (c_{.2} \text{Ecardinal}_{.2} \text{Ecardleq } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum } (\text{ty}_{.2} \text{Esum}_{.2} \text{Esum } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum } A_{.27a})) (c_{.2} \text{Epred}_{.set} \text{EUNIV } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum})) (c_{.2} \text{Epred}_{.set} \text{EUNIV } (\text{ty}_{.2} \text{Esum}_{.2} \text{Esum } \text{ty}_{.2} \text{Eenum}_{.2} \text{Eenum } A_{.27a})))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (41)$$

**Theorem 1**

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{(ty_{.2Eordinal} \cdot 2Eordinal A_{.27a})}). \\ & ((p (ap (c_{.2Epred\_set} \cdot 2Ecountable (ty_{.2Eordinal} \cdot 2Eordinal \\ & A_{.27a}) V0s)) \Rightarrow (\forall V1b \in (ty_{.2Eordinal} \cdot 2Eordinal A_{.27a}). \\ & ((p (ap (ap (c_{.2Eordinal} \cdot 2Eordlt A_{.27a}) V1b) (ap (c_{.2Eordinal} \cdot 2Esup \\ & A_{.27a}) V0s))) \Leftrightarrow (\exists V2d \in (ty_{.2Eordinal} \cdot 2Eordinal A_{.27a}). \\ & ((p (ap (ap (c_{.2Ebool} \cdot 2EIN (ty_{.2Eordinal} \cdot 2Eordinal A_{.27a}) V2d) \\ & V0s)) \wedge (p (ap (ap (c_{.2Eordinal} \cdot 2Eordlt A_{.27a}) V1b) V2d)))))))))) \end{aligned}$$