

thm_2Eordinal_2Eepsilon0_least_fixpoint
(TMbJsL3fWrsAn6mvGGJ7Lz14qXwLLZj5RHa)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x)$

Definition 3 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P x) \text{ then } (\lambda x. x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 4 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } V 0P (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `c_2Enum_2EZERO__REP` : ι be given. Assume the following.

$$\text{c_2Enum_2EZERO_REP} \in \text{omega} \tag{2}$$

Let `c_2Enum_2EABS__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EABS_num} \in (\text{ty_2Enum_2Enum}^{\text{omega}}) \tag{3}$$

Definition 5 We define `c_2Enum_2E0` to be $(\text{ap } \text{c_2Enum_2EABS_num } \text{c_2Enum_2EZERO_REP})$.

Definition 6 We define `c_2Earithmetic_2EZERO` to be `c_2Enum_2E0`.

Let `c_2Enum_2EREP__num` : ι be given. Assume the following.

$$\text{c_2Enum_2EREP_num} \in (\text{omega}^{\text{ty_2Enum_2Enum}}) \tag{4}$$

Let `c_2Enum_2ESUC__REP` : ι be given. Assume the following.

$$\text{c_2Enum_2ESUC_REP} \in (\text{omega}^{\text{omega}}) \tag{5}$$

Definition 7 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E_2B$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (7)$$

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EfromNat\ A_27a \in (ty_2Eordinal_2Eordinal\ A_27a)^{ty_2Enum_2Enum} \quad (8)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (9)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_2REP_2CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (11)$$

Definition 11 We define $c_2Eordinal_2Eordinal_2REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let $c_2Ewellorder_2Ewellorder_2REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (13)$$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (14)$$

Definition 16 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_7E$

Definition 17 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (15)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (16)$$

Definition 18 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (17)$$

Definition 19 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 20 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 21 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder A_27a)^{(2^{(ty_2Epair_2Eprod A_27a A_27a)})}) \end{aligned} \quad (18)$$

Definition 22 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Eiseg$

Definition 23 We define $c_Eset_relation_Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a)})$

Definition 24 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a)})$

Definition 25 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21 2) (\lambda V2t \in 2))))$

Definition 26 We define $c_Epred_set_EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_Epred_set_E_21 2) (\lambda V2t \in 2)))$

Definition 27 We define $c_Ewellorder_EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 28 We define $c_Ewellorder_Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 29 We define $c_Ewellorder_Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 30 We define $c_Eordinal_Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a)$

Definition 31 We define $c_Eordinal_Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 32 We define $c_Eordinal_Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal A_27a)$

Definition 33 We define $c_Epred_set_EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27b})$

Definition 34 We define $c_Epred_set_EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_Epred_set_E_21 2) (\lambda V2t \in 2)))$

Definition 35 We define $c_Eordinal_Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 36 We define $c_Eordinal_Eomega$ to be $\lambda A_27a : \iota.(ap (c_Eordinal_Esup A_27a) (ap (c_Epred_set_EIMAGE A_27a) (\lambda V0f \in (A_27a^{A_27a}).\lambda V1s \in (2^{A_27a}))))$

Let $c_Eordinal_2EordEXP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Eordinal_2EordEXP A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}(ty_2Eordinal_2Eordinal A_27a)) \quad (19)$$

Definition 37 We define $c_Eordinal_2Eepsilon0$ to be $\lambda A_27a : \iota.(ap (c_Eordinal_2Eoleast A_27a) (\lambda V0x \in 2))$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (22) \end{aligned}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal A_27a). (\forall V1a \in (ty_2Eordinal_2Eordinal A_27a). ((\neg(p (ap (ap (c_2Eordinal_2Eordlt A_27a) V0b) V1a))) \Leftrightarrow ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V1a) V0b)) \vee (V1a = V0b)))))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in (2^{(ty_2Eordinal_2Eordinal A_27a)}). (\forall V1P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}). (((\exists V2a \in (ty_2Eordinal_2Eordinal A_27a). (p (ap V1P V2a))) \wedge (\forall V3a \in (ty_2Eordinal_2Eordinal A_27a). (((\forall V4b \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V4b) V3a)) \Rightarrow (\neg(p (ap V1P V4b)))))) \wedge (p (ap V1P V3a))) \Rightarrow (p (ap V0Q V3a)))))) \Rightarrow (p (ap V0Q (ap (c_2Eordinal_2Eoleast A_27a) V1P)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ V0n))\ (c_2Eordinal_2Eomega\ A.27a)))) \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0y \in (ty_2Eordinal_2Eordinal\ A.27a). (\forall V1x \in (ty_2Eordinal_2Eordinal\ A.27a). (\forall V2a \in \\ (ty_2Eordinal_2Eordinal\ A.27a). (((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ V2a)) \wedge \\ (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ V1x)\ V0y))) \Rightarrow (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordEXP\ A.27a)\ V2a)\ V1x))\ (ap\ (ap\ (\\ c_2Eordinal_2EordEXP\ A.27a)\ V2a)\ V0y))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal\ A.27a). (\forall V1x \in (ty_2Eordinal_2Eordinal\ A.27a). ((p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\ V0a)) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordEXP\ A.27a)\ V0a)\ V1x))\ V1x)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (ap\ (c_2Eordinal_2EordEXP\ A.27a)\ (c_2Eordinal_2Eomega\ A.27a))\ (c_2Eordinal_2Epsilon0\ A.27a)) = (c_2Eordinal_2Epsilon0\ A.27a)) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (49)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in (ty_{.2}Eordinal_{.2}Eordinal \\ & A_{.27a}). ((p (ap (ap (c_{.2}Eordinal_{.2}Eordlt } A_{.27a}) V0a) (c_{.2}Eordinal_{.2}Epsilon0 \\ & A_{.27a}))) \Rightarrow ((p (ap (ap (c_{.2}Eordinal_{.2}Eordlt } A_{.27a}) V0a) (ap (ap (\\ & c_{.2}Eordinal_{.2}EordEXP } A_{.27a}) (c_{.2}Eordinal_{.2}Eomega } A_{.27a})) V0a))) \wedge \\ & (p (ap (ap (c_{.2}Eordinal_{.2}Eordlt } A_{.27a}) (ap (ap (c_{.2}Eordinal_{.2}EordEXP } \\ & A_{.27a}) (c_{.2}Eordinal_{.2}Eomega } A_{.27a})) V0a)) (c_{.2}Eordinal_{.2}Epsilon0 \\ & A_{.27a})))))) \end{aligned}$$