

thm_2Eordinal_2Efixpoints__exist
(TMR3YrPJ8Cvj4PxbDZkSXU2Pe6nN5ZWodaU)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num ($

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 5 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Earithmetic_2EFUNPOW : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda a.nonempty\ A.\lambda a \Rightarrow c_2Earithmetic_2EFUNPOW\ A.\lambda a \in ((A.\lambda a^{A-27a})^{ty_2Enum_2Enum})^{(A.\lambda a^{A-27a})} \tag{6}$$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 7 We define `c_2Ecombin_2EC` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0 f \in ((A. 27c^{A. 27b})^{A. 27a}))$

Definition 8 We define `c_2Ecombin_2EK` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. (\lambda V0 x \in A. 27a. (\lambda V1 y \in A. 27b. V0 x))$

Definition 9 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0 f \in ((A. 27c^{A. 27b})^{A. 27a}))$

Definition 10 We define `c_2Ecombin_2EI` to be $\lambda A. 27a : \iota. (\text{ap } (\text{ap } (\text{c_2Ecombin_2ES } A. 27a \ (A. 27a^{A. 27a})) \ A$

Definition 11 We define `c_2Ecombin_2Eo` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0 f \in (A. 27b^{A. 27c}). \lambda V1$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 \ A1) \quad (7)$$

Let `ty_2Ewellorder_2Ewellorder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ewellorder_2Ewellorder } A0) \quad (8)$$

Let `ty_2Eordinal_2Eordinal` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eordinal_2Eordinal } A0) \quad (9)$$

Let `c_2Eordinal_2Eordinal__REP__CLASS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$A. 27a \in ((2^{(\text{ty_2Ewellorder_2Ewellorder } (\text{ty_2Esum_2Esum } \text{ty_2Enum_2Enum } A. 27a))} (\text{ty_2Eordinal_2Eordinal } A. 27a))) \quad (10)$$

Definition 12 We define `c_2Eordinal_2Eordinal__REP` to be $\lambda A. 27a : \iota. \lambda V0 a \in (\text{ty_2Eordinal_2Eordinal } A$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A0 \ A1) \quad (11)$$

Let `c_2Ewellorder_2Ewellorder__REP` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$A. 27a \in ((2^{(\text{ty_2Epair_2Eprod } A. 27a \ A. 27a)} (\text{ty_2Ewellorder_2Ewellorder } A. 27a))) \quad (12)$$

Definition 13 We define `c_2Ebool_2E_21` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0 t \in 2. V0 t)).$

Definition 14 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \ P \Rightarrow \ p \ Q)$ of type ι .

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.(ap (c_2Ebool_2E_7E) (c_2Ebool_2E_21 2) V2t) V2t) V1t2) V0t1))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (13)$$

Definition 17 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_2F_5C) V0x V1y)$

Definition 18 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (14)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (15)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (16)$$

Definition 20 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 21 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a A_27a)$

Definition 22 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS A_27a \in ((ty_2Ewellorder_2Ewellorder A_27a)^{(2^{(ty_2Epair_2Eprod A_27a A_27a)})}) \quad (17)$$

Definition 23 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder A_27a A_27a)$

Definition 24 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) (c_2Ebool_2E_7E) (c_2Ebool_2E_7E) V0P) V0P))$

Definition 25 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 26 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod$

Definition 27 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2) (\lambda V2t$

Definition 28 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 29 We define $c_Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A$

Definition 30 We define $c_Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 31 We define $c_Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 32 We define $c_Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a).\lambda$

Definition 33 We define $c_Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal A_27a).($

Definition 34 We define $c_Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 35 We define $c_Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_Epred_s$

Definition 36 We define $c_Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).(ap$

Definition 37 We define $c_Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).($

Definition 38 We define $c_Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^A$

Definition 39 We define $c_Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^A$

Definition 40 We define $c_Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2EF).$

Definition 41 We define $c_Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2ET).$

Definition 42 We define $c_Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in ($

Let $c_Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Esum_2EABS_sum \\ A_27a A_27b \in ((ty_2Esum_2Esum A_27a A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (18)$$

Definition 43 We define c_Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap (c_Esum_2EABS$

Definition 44 We define c_Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_Esum_2EABS$

Let $c_Esum_2Esum_CASE : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\ nonempty A_27c \Rightarrow c_Esum_2Esum_CASE A_27a A_27b A_27c \in (((A_27c^{(A_27c^{A_27b})})^{(A_27c^{A_27a})})^{(ty_2E$$
 \end{aligned} \quad (19)

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0f \in (A_27a^{A_27a}).(\forall V1x \in \\ A_27a.((ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW\ A_27a)\ V0f)\ c_2Enum_2E0) \\ V1x) = V1x))) \wedge (\forall V2f \in (A_27a^{A_27a}).(\forall V3n \in ty_2Enum_2Enum. \\ (\forall V4x \in A_27a.((ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW\ A_27a) \\ V2f)\ (ap\ c_2Enum_2ESUC\ V3n))\ V4x) = (ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW \\ A_27a)\ V2f)\ V3n)\ (ap\ V2f\ V4x)))))))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0f \in (A_27a^{A_27a}).(\forall V1n \in \\ ty_2Enum_2Enum.(\forall V2x \in A_27a.((ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW \\ A_27a)\ V0f)\ (ap\ c_2Enum_2ESUC\ V1n))\ V2x) = (ap\ V0f\ (ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW \\ A_27a)\ V0f)\ V1n)\ V2x)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & ((\forall V0f \in (A_27a^{A_27a}).(\forall V1x \in \\ A_27a.((ap\ (ap\ (ap\ (c_2Earithmic_2EFUNPOW\ A_27a)\ V0f)\ c_2Enum_2E0) \\ V1x) = V1x))) \end{aligned} \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (26)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \end{aligned} \quad (28)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (30)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (33)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A_27a.(p (ap V1Q V4x))))))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p (ap V0P V3x)) \wedge (p V1Q))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.((\exists V2x \in A_27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a.(p (ap V0P V3x))) \wedge (p V1Q))))) \quad (38)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\exists V2x \in A.27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a. (p (ap V1Q V3x))))))) \quad (39)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \Rightarrow (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a. (p (ap V0P V3x)) \Rightarrow (p V1Q)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (43)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (44)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in 2. (((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1a \in A.27a. ((\exists V2x \in A.27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0f \in (2^{A.27a}). (\forall V1v \in A.27a. ((\forall V2x \in A.27a. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (47)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27a}). (\forall V1s \in (2^{A_27a}). \\
& \quad (\forall V2t \in (2^{A_27b}). ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a \\
& \quad A_27b)\ V1s)\ V2t))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27c\ A_27b) \\
& \quad (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27c)\ V0f)\ V1s))\ V2t))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI \\
A_27a)\ V0x) = V0x)) \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (((ap\ (ap\ (c_2Ecombin_2Eo\ A_27a\ A_27b \\
& \quad A_27b)\ (c_2Ecombin_2EI\ A_27b))\ V0f) = V0f) \wedge ((ap\ (ap\ (c_2Ecombin_2Eo \\
& \quad A_27a\ A_27b\ A_27a)\ V0f)\ (c_2Ecombin_2EI\ A_27a)) = V0f)))
\end{aligned} \tag{50}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Eordinal_2Eordinal \\
A_27a). (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V0w)\ V0w)))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a). (\forall V1s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). ((\\
& \quad (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal\ A_27a) \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V1s)\ (c_2Epred_set_2EUNIV \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \wedge (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Eordinal_2Eordinal\ A_27a)\ V0b)\ V1s))) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& \quad A_27a)\ (ap\ (c_2Eordinal_2Esup\ A_27a)\ V1s))\ V0b))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s1 \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\
& \quad (\forall V1s2 \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (((p\ (ap\ (\\
& \quad \quad ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal\ A_27a)\ (\\
& \quad \quad ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V0s1)\ (c_2Epred_set_2EUNIV \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \wedge ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\
& \quad (ty_2Eordinal_2Eordinal\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\
& \quad A_27a))\ V1s2)\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\
& \quad A_27a)))) \wedge ((\forall V2a \in (ty_2Eordinal_2Eordinal\ A_27a). ((\\
& \quad p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal\ A_27a)\ V2a) \\
& \quad V0s1)) \Rightarrow (\exists V3b \in (ty_2Eordinal_2Eordinal\ A_27a). ((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal\ A_27a)\ V3b)\ V1s2)) \wedge \\
& \quad (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V3b)\ V2a)))))) \wedge (\forall V4b \in \\
& \quad (ty_2Eordinal_2Eordinal\ A_27a). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal \\
& \quad A_27a)\ V4b)\ V1s2)) \Rightarrow (\exists V5a \in (ty_2Eordinal_2Eordinal\ A_27a). \\
& \quad ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal\ A_27a)\ V5a) \\
& \quad V0s1)) \wedge (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V5a)\ V4b))))))))) \Rightarrow \\
& \quad ((ap\ (c_2Eordinal_2Esup\ A_27a)\ V0s1) = (ap\ (c_2Eordinal_2Esup \\
& \quad A_27a)\ V1s2))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\
& \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\
& \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\
& \quad (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\
& \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\
& \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (ap \\
& \quad (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))) \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b.(\forall V1s \in (2^{A_27a}).(\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27b)\ V2f)\ V1s)))) \Leftrightarrow (\exists V3x \in A_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1s \in (2^{A_27a}).(\forall V2t \in \\ & \quad (2^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2ESURJ\ A_27a\ A_27b)\ V0f)\ V1s)\ V2t)) \Rightarrow (\exists V3g \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ\ A_27b\ A_27a)\ V3g)\ V2t)\ V1s)) \wedge (\forall V4y \in A_27b.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V4y)\ V2t)) \Rightarrow ((ap\ V0f\ (ap\ V3g\ V4y)) = V4y))))))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee \neg(p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee \neg(p V0p))))))))))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge (\neg(p V1q) \vee \neg(p V0p))))))
\end{aligned} \tag{69}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow (p V0p))) \tag{70}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q)) \Rightarrow \neg(p V1q))) \tag{71}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q)) \Rightarrow \neg(p V0p))) \tag{72}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \vee (p V1q)) \Rightarrow \neg(p V1q))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2. (\neg(\neg(p V0p)) \Rightarrow (p V0p))) \tag{74}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0P \in (2^{(ty_2Esum_2Esum A_27a A_27b)}). ((\forall V1s \in \\
& (ty_2Esum_2Esum A_27a A_27b). (p (ap V0P V1s))) \Leftrightarrow ((\forall V2x \in \\
& A_27a. (p (ap V0P (ap (c_2Esum_2EINL A_27a A_27b) V2x)))) \wedge (\forall V3y \in \\
& A_27b. (p (ap V0P (ap (c_2Esum_2EINR A_27a A_27b) V3y))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\
& nonempty A_27c \Rightarrow ((\forall V0x \in A_27a. (\forall V1f \in (A_27c^{A_27a}). \\
& (\forall V2f1 \in (A_27c^{A_27b}). ((ap (ap (ap (c_2Esum_2Esum_CASE \\
& A_27a A_27b A_27c) (ap (c_2Esum_2EINL A_27a A_27b) V0x)) V1f) V2f1) = \\
& (ap V1f V0x)))) \wedge (\forall V3y \in A_27b. (\forall V4f \in (A_27c^{A_27a}). \\
& (\forall V5f1 \in (A_27c^{A_27b}). ((ap (ap (ap (c_2Esum_2Esum_CASE \\
& A_27a A_27b A_27c) (ap (c_2Esum_2EINR A_27a A_27b) V3y)) V4f) V5f1) = \\
& (ap V5f1 V3y))))))
\end{aligned} \tag{76}$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal \\ & A_{.27a})(ty_2Eordinal_2Eordinal\ A_{.27a}).((\forall V1s \in (2^{(ty_2Eordinal_2Eordinal\ A_{.27a})}). \\ & (((\neg(V1s = (c_2Epred_set_2EEMPTY\ (ty_2Eordinal_2Eordinal\ A_{.27a})))) \wedge \\ & (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal\ A_{.27a}) \\ & (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_{.27a}))\ V1s)\ (c_2Epred_set_2EUNIV \\ & (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_{.27a})))))) \Rightarrow ((ap\ V0f\ (ap\ (c_2Eordinal_2Esup \\ & A_{.27a})\ V1s)) = (ap\ (c_2Eordinal_2Esup\ A_{.27a})\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & (ty_2Eordinal_2Eordinal\ A_{.27a})\ (ty_2Eordinal_2Eordinal\ A_{.27a})) \\ & V0f)\ V1s)))))) \wedge (\forall V2x \in (ty_2Eordinal_2Eordinal\ A_{.27a}). \\ & (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ (ap\ V0f\ V2x))\ V2x)))))) \Rightarrow \\ & (\forall V3a \in (ty_2Eordinal_2Eordinal\ A_{.27a}).(\exists V4b \in (\\ & ty_2Eordinal_2Eordinal\ A_{.27a}).(\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ & A_{.27a})\ V4b)\ V3a)))) \wedge ((ap\ V0f\ V4b) = V4b)))))) \end{aligned}$$