thm_2Eordinal_2EfromNat__def__compute (TMG5EhMSUDBjKx1WEsb4kXhEps1Jxt3FVeQ)

October 26, 2020

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 6 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 7 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$*.*

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{8}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{9}$$

Let $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \tag{10}$$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \tag{11}$$

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota \Rightarrow \iota$*.*

Definition 11 *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{12}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{13}$$

Definition 12 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$*.*

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{14}$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 17 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{15}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{16}$$

Definition 18 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{17}$$

Definition 19 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 20 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 21 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{18}$$

Definition 22 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a:\iota.(\lambda V0P\in(2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 24 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A}$

Definition 25 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod}$

Definition 26 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1\in 2.(\lambda V1t2\in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t\in$

Definition 27 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c.$

Definition 28 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Ewellorder\_2Ewellorder\ A$

Definition 29 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1\in(ty\_2Ewellorder\_2$

Definition 30 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1\in(ty\_2Ewellorder\_2E$

Definition 31 *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a:\iota.\lambda V0T1\in(ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

Definition 32 *We define* c_2Eordinal_2Eoleast *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap$

Definition 33 *We define* c_2Eordinal_2EordSUC *to be* $\lambda A\_27a:\iota.\lambda V0a\in(ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let $c\_2Eordinal\_2EfromNat:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a\in((ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum})\quad(19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f\in((A\_27a^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}).(\forall V1g\in(A\_27a^{ty\_2Enum\_2Enum}).((\forall V2n\in ty\_2Enum\_2Enum.((ap\ V1g\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))=(ap\ (ap\ V0f\ V2n)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))))\Leftrightarrow((\forall V3n\in ty\_2Enum\_2Enum.((ap\ V1g\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))=(ap\ (ap\ V0f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V3n)))))\wedge(\forall V4n\in ty\_2Enum\_2Enum.((ap\ V1g\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V4n)))=(ap\ (ap\ V0f\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V4n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V4n)))))))))\quad(20)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})=(ap\ (\mathsf{c\_2Eordinal\_2Eoleast}\ A\_27a)\ (\lambda V0a\in(ty\_2Eordinal\_2Eordinal\ A\_27a).\mathsf{c\_2Ebool\_2ET})))\wedge(\forall V1n\in ty\_2Enum\_2Enum.((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))=(ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ V1n)))))\quad(21)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}) = (ap\ (\mathsf{c\_2Eordinal\_2Eoleast}\ A\_27a)\ (\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).\mathsf{c\_2Ebool\_2ET})))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1n))) = (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V1n)))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))))\wedge(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V2n))) = (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V2n))))))))$$