

# thm\_2Eordinal\_2EfromNat\_lt\_epsilon0 (TM- LoLk1DvfFNFduLbNyBuFThLbB9UPbn8eT)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let  $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{4}$$

Let  $c\_2Ewellorder\_2Ewellorder\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) (ty\_2Ewellorder\_2Ewellorder\ A\_27a)) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E21\ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS\_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

**Definition 8** We define `c_2Epair_2E_2C` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E_2C))$

**Definition 9** We define `c_2Ebool_2EIN` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let `c_2Epair_2ESND` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let `c_2Epair_2EFST` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

**Definition 10** We define `c_2Epair_2EUNCURRY` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let `c_2Epred_set_2EGSPEC` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred\_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

**Definition 11** We define `c_2Eset_relation_2Estrict` to be  $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

**Definition 12** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) p)$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40))))$

**Definition 14** We define `c_2Eset_relation_2Erangle` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

**Definition 15** We define `c_2Eset_relation_2Edomain` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

**Definition 16** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

**Definition 17** We define `c_2Epred_set_2EUNION` to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred\_set_2EUNION))$

**Definition 18** We define `c_2Ewellorder_2EelsOf` to be  $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

**Definition 19** We define  $c\_Ewellorder\_Eorderiso$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder A\_27a A\_27b)$

Let  $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Eordinal\_2Eordinal A0) \quad (10)$$

Let  $c\_2Eordinal\_2Eordinal\_ABS\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_ABS\_CLASS A\_27a \in ((ty\_2Eordinal\_2Eordinal A\_27a)^{(2^{(ty\_2Ewellorder\_2Ewellorder (ty\_2Esum\_2Esum ty\_2Enum\_2Enum A\_27a))})}) \quad (11)$$

**Definition 20** We define  $c\_2Eordinal\_2Eordinal\_ABS$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (ty\_2Ewellorder\_2Ewellorder A\_27a A\_27a)$

Let  $c\_2Eordinal\_2Eordinal\_REP\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_REP\_CLASS A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder (ty\_2Esum\_2Esum ty\_2Enum\_2Enum A\_27a))})^{(ty\_2Eordinal\_2Eordinal A\_27a)}) \quad (12)$$

**Definition 21** We define  $c\_2Eordinal\_2Eordinal\_REP$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Eordinal\_2Eordinal A\_27a A\_27a)$

Let  $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eordinal\_2EfromNat A\_27a \in (ty\_2Eordinal\_2Eordinal A\_27a)^{ty\_2Enum\_2Enum} \quad (13)$$

**Definition 22** We define  $c\_2Ewellorder\_2Eiseg$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Ewellorder\_2Ewellorder A\_27a A\_27a)$

**Definition 23** We define  $c\_2Eset\_relation\_2Errestrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})$

Let  $c\_2Ewellorder\_2Ewellorder\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_ABS A\_27a \in ((ty\_2Ewellorder\_2Ewellorder A\_27a)^{(2^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \quad (14)$$

**Definition 24** We define  $c\_2Ewellorder\_2Ewobound$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1w \in (ty\_2Ewellorder\_2Ewellorder A\_27a A\_27a)$

**Definition 25** We define  $c\_2Ewellorder\_2Eorderlt$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder A\_27a A\_27b)$

**Definition 26** We define  $c\_2Eordinal\_2Eordlt$  to be  $\lambda A\_27a : \iota. \lambda V0T1 \in (ty\_2Eordinal\_2Eordinal A\_27a A\_27a)$

**Definition 27** We define  $c\_2Eordinal\_2Epreds$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Eordinal\_2Eordinal A\_27a A\_27a)$

**Definition 28** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (ty\_2Eordinal\_2Eordinal A\_27a A\_27a)$

**Definition 29** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap (c\_2Epred\_set\_2EIMAGE P))$

**Definition 30** We define  $c\_2Eordinal\_2Eoleast$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).$  ( $ap$

**Definition 31** We define  $c\_2Eordinal\_2Esup$  to be  $\lambda A\_27a : \iota. \lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).$  ( $ap$

**Definition 32** We define  $c\_2Eordinal\_2Eomega$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eordinal\_2Esup\ A\_27a) (ap (c\_2Epr$

Let  $c\_2Eordinal\_2EordEXP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EordEXP\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \quad (15)$$

**Definition 33** We define  $c\_2Eordinal\_2Eepsilon0$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eordinal\_2Eoleast\ A\_27a) (\lambda V0x \in$

**Definition 34** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0x \in A\_27a. (\lambda V1y \in A\_27b. V0x)$

**Definition 35** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. (\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 36** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota. (ap (ap (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A$

**Definition 37** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda A\_27d : \iota. \lambda V0f$

**Definition 38** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R1 \in ((2^{A\_27a})^{A\_27b})^{A\_27c}$

**Definition 39** We define  $c\_2Equotient\_2EQUOTIENT$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0R \in ((2^{A\_27a})^{A\_27b})^{A\_27c}$

**Definition 40** We define  $c\_2Ecombin\_2EW$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27c}). (\lambda V1x$

**Definition 41** We define  $c\_2Equotient\_2Erespects$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (c\_2Ecombin\_2EW\ A\_27a\ A\_27b)$

**Definition 42** We define  $c\_2Ebool\_2ERES\_FORALL$  to be  $\lambda A\_27a : \iota. (\lambda V0p \in (2^{A\_27a}). (\lambda V1m \in (2^{A\_27a}).$

**Definition 43** We define  $c\_2Equotient\_2EEQUIV$  to be  $\lambda A\_27a : \iota. \lambda V0E \in ((2^{A\_27a})^{A\_27b}). (ap (c\_2Ebool\_2E$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ & (p\ V0t) \Rightarrow False) \Leftrightarrow \neg (p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (18)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ \forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = \\ V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ p\ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p \\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ ((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap\ (c\_2Ecombin\_2EI \\ A\_27a)\ V0x) = V0x)) \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ (ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum \\ A\_27a))\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (c\_2Ewellorder\_2Eorderiso \\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum \\ A\_27a))\ (c\_2Eordinal\_2Eordinal\_ABS\ A\_27a))\ (c\_2Eordinal\_2Eordinal\_REP \\ A\_27a))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum. ( \\ p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat \\ A\_27a)\ V0n))\ (c\_2Eordinal\_2Eomega\ A\_27a)))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt \\ A\_27a)\ (c\_2Eordinal\_2Eomega\ A\_27a))\ (c\_2Eordinal\_2Eepsilon0 \\ A\_27a))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27a\ A\_27a)\ (c\_2Emin\_2E\_3D\ A\_27a))\ (c\_2Ecombin\_2EI\ A\_27a))\ ( \\ & c\_2Ecombin\_2EI\ A\_27a))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\ & (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\ & (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\ & V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\ & (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E \\ & A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27c \\ & A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\ & A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2)))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\ & (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\ & (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\ & V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\ & (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\ & ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\ & A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\ & (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\ & (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\ & A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\ & A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\ & (ap\ V1abs\ V4x2)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2ERespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f))))))))) \\
& \hspace{15em} (33)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in \\
& \quad (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2ERespects\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2ERespects \\
& \quad A\_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in \\
& \quad A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (c\_2Equotient\_2EEQUIV\ A\_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2ERespects \\
& \quad A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E\_21\ A\_27a)\ V1P)))))) \\
& \hspace{15em} (36)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{15em} (37)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (47)$$



Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (48)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27a)\ V0w)\ V0w))) \quad (49)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow ( \forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b). ((p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \Rightarrow (p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27b\ A\_27a)\ V1w2)\ V0w1)))))) \quad (50)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow \forall A\_27c. \text{nonempty } A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b). (\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c). (((p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \wedge (p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \quad (51)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow \forall A\_27c. \text{nonempty } A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b). (\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c). (((p (ap (ap (c\_2Ewellorder\_2Eorderlt\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \wedge (p (ap (ap (c\_2Ewellorder\_2Eorderlt\ A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p (ap (ap (c\_2Ewellorder\_2Eorderlt\ A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \quad (52)$$

Assume the following.

$$\forall A\_27a. \text{nonempty } A\_27a \Rightarrow \forall A\_27b. \text{nonempty } A\_27b \Rightarrow \forall A\_27c. \text{nonempty } A\_27c \Rightarrow \forall A\_27d. \text{nonempty } A\_27d \Rightarrow (\forall V0x0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). (\forall V1y0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b). (\forall V2a0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c). (\forall V3b0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27d). (((p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27b)\ V0x0)\ V1y0)) \wedge (p (ap (ap (c\_2Ewellorder\_2Eorderiso\ A\_27c\ A\_27d)\ V2a0)\ V3b0))) \Rightarrow ((p (ap (ap (c\_2Ewellorder\_2Eorderlt\ A\_27a\ A\_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p (ap (ap (c\_2Ewellorder\_2Eorderlt\ A\_27b\ A\_27d)\ V1y0)\ V3b0)))))) \quad (53)$$

**Theorem 1**

$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0n \in \text{ty\_2Enum\_2Enum.}$   
 $p \text{ (ap (ap (c\_2Ordinal\_2Eordlt } A_{27a}) \text{ (ap (c\_2Ordinal\_2EfromNat}$   
 $A_{27a}) V0n)) \text{ (c\_2Ordinal\_2Epsilon0 } A_{27a}))))$