

thm_2Eordinal_2EordADD_CANCEL1
(TMWG4YUGjss8tpEoWcmfnVX1dr9E41jEhqc)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \tag{4}$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) (ty_2Ewellorder_2Ewellorder\ A_27a)) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E_2C))$

Definition 9 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

Definition 10 We define `c_2Epair_2EUNCURRY` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

Definition 11 We define `c_2Eset_relation_2Estrict` to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 12 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) p)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40))))$

Definition 14 We define `c_2Eset_relation_2Erange` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 15 We define `c_2Eset_relation_2Edomain` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 16 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

Definition 17 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred_set_2EUNION))$

Definition 18 We define `c_2Ewellorder_2EelsOf` to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 19 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})}) \quad (11)$$

Definition 20 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ewellorder_2Ewellorder$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS\ A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (12)$$

Definition 21 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A$

Definition 22 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 23 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a$

Definition 24 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (13)$$

Definition 25 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder$

Definition 26 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 27 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a).\lambda$

Definition 28 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a).(c$

Definition 29 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_s$

Definition 30 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).(ap$

Definition 31 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).(c$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (14)$$

Definition 32 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (15)$$

Definition 33 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (16)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (17)$$

Definition 34 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ 0)$

Definition 35 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2Ebool_2Ebool_2Ebool)$

Definition 36 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ x)\ s)$

Definition 37 We define $c_2Eset_relation_2Emaximal_elements$ to be $\lambda A_27a : \iota. \lambda V0xs \in (2^{A_27a}). \lambda V1r \in (2^{A_27a}). (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ x)\ r)$

Definition 38 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ e)$

Definition 39 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ x)$

Definition 40 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ t1)\ t2))))$

Definition 41 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap\ (ap\ (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ x)\ P)\ 0))$

Definition 42 We define $c_2Eordinal_2Eomax$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ s))$

Definition 43 We define $c_2Eordinal_2EordSUC$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a). (ap\ (c_2Ebool_2Ebool_2Ebool_2Ebool\ a))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 44 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ordinal_2EfromNat\ A_27a \in ((ty_2Ordinal_2Ordinal\ A_27a)^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Ordinal_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ordinal_2EordADD\ A_27a \in (((ty_2Ordinal_2Ordinal\ A_27a)^{(ty_2Ordinal_2Ordinal\ A_27a)})^{(ty_2Ordinal_2Ordinal\ A_27a)}) \quad (21)$$

Definition 45 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 46 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 47 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a}))\ A_27a))$

Definition 48 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0j$

Definition 49 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 50 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27b})$

Definition 51 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in ((A_27b^{A_27a})^{A_27a}). (\lambda V1x$

Definition 52 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (c_2Ecombin_2EW\ A_27a\ A_27b)$

Definition 53 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}).$

Definition 54 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota. \lambda V0E \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2ERES_FORALL$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. ((p\ V0t) \vee (\neg(p\ V0t)))) \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Leftrightarrow (p \ V1t2)) \Leftrightarrow (((p \\ & V0t1) \Rightarrow (p \ V1t2)) \wedge ((p \ V1t2) \Rightarrow (p \ V0t1)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\ & 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((ap \ (c.2Ecombin_2EI \ A_27a) \ V0x) = V0x)) \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (p\ (ap\ (ap\ (ap\ (c.2Equotient.2EQUOTIENT \\ & (ty.2Ewellorder.2Ewellorder\ (ty.2Esum.2Esum\ ty.2Enum.2Enum \\ & A.27a))\ (ty.2Eordinal.2Eordinal\ A.27a))\ (c.2Ewellorder.2Eorderiso \\ & (ty.2Esum.2Esum\ ty.2Enum.2Enum\ A.27a)\ (ty.2Esum.2Esum\ ty.2Enum.2Enum \\ & A.27a)))\ (c.2Eordinal.2Eordinal_ABS\ A.27a))\ (c.2Eordinal.2Eordinal_REP \\ & A.27a))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0x \in (ty.2Eordinal.2Eordinal \\ & A.27a).((\neg(p\ (ap\ (ap\ (c.2Eordinal.2Eordlt\ A.27a)\ (ap\ (c.2Eordinal.2EfromNat \\ & A.27a)\ c.2Enum.2E0))\ V0x))) \Leftrightarrow (V0x = (ap\ (c.2Eordinal.2EfromNat \\ & A.27a)\ c.2Enum.2E0)))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0x \in ty.2Enum.2Enum. (\\ & \forall V1y \in ty.2Enum.2Enum. (((ap\ (c.2Eordinal.2EfromNat\ A.27a) \\ & V0x) = (ap\ (c.2Eordinal.2EfromNat\ A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0b \in (ty.2Eordinal.2Eordinal \\ & A.27a).(((ap\ (ap\ (c.2Eordinal.2EordADD\ A.27a)\ V0b)\ (ap\ (c.2Eordinal.2EfromNat \\ & A.27a)\ c.2Enum.2E0)) = V0b) \wedge ((\forall V1a \in (ty.2Eordinal.2Eordinal \\ & A.27a).((ap\ (ap\ (c.2Eordinal.2EordADD\ A.27a)\ V0b)\ (ap\ (c.2Eordinal.2EordSUC \\ & A.27a)\ V1a)) = (ap\ (c.2Eordinal.2EordSUC\ A.27a)\ (ap\ (ap\ (c.2Eordinal.2EordADD \\ & A.27a)\ V0b)\ V1a)))) \wedge (\forall V2a \in (ty.2Eordinal.2Eordinal\ A.27a). \\ & (((p\ (ap\ (ap\ (c.2Eordinal.2Eordlt\ A.27a)\ (ap\ (c.2Eordinal.2EfromNat \\ & A.27a)\ c.2Enum.2E0))\ V2a)) \wedge ((ap\ (c.2Eordinal.2Eomax\ A.27a)\ (\\ & ap\ (c.2Eordinal.2Epreds\ A.27a)\ V2a)) = (c.2Eoption.2ENONE\ (ty.2Eordinal.2Eordinal \\ & A.27a)))))) \Rightarrow ((ap\ (ap\ (c.2Eordinal.2EordADD\ A.27a)\ V0b)\ V2a) = (ap \\ & (c.2Eordinal.2Esup\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ (ty.2Eordinal.2Eordinal \\ & A.27a)\ (ty.2Eordinal.2Eordinal\ A.27a))\ (ap\ (c.2Eordinal.2EordADD \\ & A.27a)\ V0b))\ (ap\ (c.2Eordinal.2Epreds\ A.27a)\ V2a)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow & (\forall V0b \in (ty.2Eordinal.2Eordinal \\ & A.27a).(\forall V1a \in (ty.2Eordinal.2Eordinal\ A.27a).((p\ (ap \\ & (ap\ (c.2Eordinal.2Eordlt\ A.27a)\ V1a)\ (ap\ (ap\ (c.2Eordinal.2EordADD \\ & A.27a)\ V1a)\ V0b))) \Leftrightarrow (p\ (ap\ (ap\ (c.2Eordinal.2Eordlt\ A.27a)\ (ap\ (\\ & c.2Eordinal.2EfromNat\ A.27a)\ c.2Enum.2E0))\ V0b)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (\\ & c_2Ecombin_2EI\ A_27a))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ & A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\ & A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ & A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2)))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\ & (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\ & (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\ & V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\ & (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\ & ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ & A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\ & (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow & \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\ & (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\ & A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\ & (ap\ V1abs\ V4x2)))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))) \\
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2ERespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2ERespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
\end{aligned} \tag{48}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (50)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (63)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a). (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27a)\ V0w)\ V0w))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b). ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27b\ A_27a)\ V1w2)\ V0w1)))))) \quad (65)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b). (\forall V2w3 \in (ty_2Ewellorder_2Ewellorder\ A_27c). (((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27b\ A_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27c)\ V0w1)\ V2w3)))))) \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a). ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27a)\ V0w)\ V0w)) \Leftrightarrow False)) \quad (67)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow \forall A_27c. \\
& \quad \text{nonempty } A_27c \Rightarrow \forall A_27d.\text{nonempty } A_27d \Rightarrow (\forall V0x0 \in (\\
& \quad \text{ty_2Ewellorder_2Ewellorder } A_27a).(\forall V1y0 \in (\text{ty_2Ewellorder_2Ewellorder} \\
& \quad A_27b).(\forall V2a0 \in (\text{ty_2Ewellorder_2Ewellorder } A_27c). (\\
& \quad \forall V3b0 \in (\text{ty_2Ewellorder_2Ewellorder } A_27d).(((p (ap (ap \\
& \quad (\text{c_2Ewellorder_2Eorderiso } A_27a } A_27b) V0x0) V1y0)) \wedge (p (ap (ap \\
& \quad (\text{c_2Ewellorder_2Eorderiso } A_27c } A_27d) V2a0) V3b0))) \Rightarrow ((p (ap \\
& \quad (ap (\text{c_2Ewellorder_2Eorderlt } A_27a } A_27c) V0x0) V2a0)) \Leftrightarrow (p (ap \\
& \quad (ap (\text{c_2Ewellorder_2Eorderlt } A_27b } A_27d) V1y0) V3b0)))))))))
\end{aligned} \tag{68}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0c \in (\text{ty_2Eordinal_2Eordinal} \\
& \quad A_27a).(\forall V1a \in (\text{ty_2Eordinal_2Eordinal } A_27a).((V1a = \\
& \quad (ap (ap (\text{c_2Eordinal_2EordADD } A_27a) V1a) V0c)) \Leftrightarrow (V0c = (ap (\text{c_2Eordinal_2EfromNat} \\
& \quad A_27a) \text{c_2Enum_2E0})))))) \wedge (\forall V2c \in (\text{ty_2Eordinal_2Eordinal} \\
& \quad A_27a).(\forall V3a \in (\text{ty_2Eordinal_2Eordinal } A_27a).(((ap (\\
& \quad \text{ap } (\text{c_2Eordinal_2EordADD } A_27a) V3a) V2c) = V3a) \Leftrightarrow (V2c = (ap (\text{c_2Eordinal_2EfromNat} \\
& \quad A_27a) \text{c_2Enum_2E0})))))))))
\end{aligned}$$