

thm_2Eordinal_2EordADD__MONO (TM- MGZ1GCd6vXfAVWwbwZ6EuJSxxeph2ew4q)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_EIN$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(\lambda p V1f V0x)))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{2}$$

Definition 7 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \tag{3}$$

Definition 8 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27b^{A_27a}).$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (4)$$

Definition 9 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone$

Definition 11 We define c_2Ebool_2E2F to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (5)$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \quad (6)$$

Definition 13 We define c_2Esum_2EINR to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27b.(ap\ (c_2Esum_2EABS$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (7)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \quad (8)$$

Definition 14 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota.(ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ ($

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (9)$$

Definition 15 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2E2F)$.

Definition 16 We define $c_2Ebool_2E5C_2E2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in$

Definition 17 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (10)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (11)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS \\ A.27a \in & ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a))})^{(ty_2Eordinal_2Eordinal\ A.27a)}) \end{aligned} \quad (12)$$

Definition 18 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A.27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A.27a)$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c_2Ewellorder_2Ewellorder_REP \\ A.27a \in & ((2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)})^{(ty_2Ewellorder_2Ewellorder\ A.27a)}) \end{aligned} \quad (13)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND \\ & A.27a\ A.27b \in (A.27b)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (14)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST \\ & A.27a\ A.27b \in (A.27a)^{(ty_2Epair_2Eprod\ A.27a\ A.27b)} \end{aligned} \quad (15)$$

Definition 19 We define $c_2Epair_2EUNCURRY$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c)^{A.27a})$

Definition 20 We define $c_2Eset_relation_2Estrict$ to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)})$

Definition 21 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A.27a)$

Definition 22 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A.27a \in & ((ty_2Ewellorder_2Ewellorder\ A.27a)^{(2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)})}) \end{aligned} \quad (16)$$

Definition 23 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1w \in (ty_2Ewellorder\ A.27a)$

Definition 24 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ A.27a))))$

Definition 25 We define $c_Eset_relation_Erange$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a)})$

Definition 26 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a)})$

Definition 27 We define $c_Epred_set_EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_Epred_set_EUNION\ A_27a\ V0s\ V1t))$

Definition 28 We define $c_Ewellorder_EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 29 We define $c_Ewellorder_Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 30 We define $c_Ewellorder_Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 31 We define $c_Eordinal_Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 32 We define $c_Eset_relation_Emaximal_elements$ to be $\lambda A_27a : \iota. \lambda V0xs \in (2^{A_27a}). \lambda V1r \in (2^{A_27a})$

Definition 33 We define c_Esum_EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_Esum_2EABS\ A_27a\ A_27b\ V0e))$

Definition 34 We define $c_Eoption_ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_Eoption_2Eoption\ A_27a\ V0x))$

Definition 35 We define c_Ebool_ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_Ebool_2ECOND\ A_27a\ V0t\ V1t1\ V2t2))))$

Definition 36 We define $c_Eoption_ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap\ (ap\ (ap\ (c_Ebool_2ECOND\ A_27a\ V0P))))$

Definition 37 We define $c_Eordinal_Eomax$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$

Definition 38 We define $c_Eordinal_Eoleast$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$

Definition 39 We define $c_Eordinal_EordSUC$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $c_Eenum_EZERO_REP : \iota$ be given. Assume the following.

$$c_Eenum_EZERO_REP \in \omega \tag{17}$$

Let $c_Eenum_EABS_num : \iota$ be given. Assume the following.

$$c_Eenum_EABS_num \in (ty_2Eenum_2Eenum^{\omega}) \tag{18}$$

Definition 40 We define c_Eenum_E0 to be $(ap\ c_Eenum_EABS_num\ c_Eenum_EZERO_REP)$.

Let $c_Eordinal_EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Eordinal_EfromNat\ A_27a \in (ty_2Eordinal_2Eordinal\ A_27a)^{ty_2Eenum_2Eenum} \tag{19}$$

Let $c_Eordinal_EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_Eordinal_EordADD\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \tag{20}$$

Definition 41 We define $c_Eordinal_Epreds$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 42 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EBIGUNION) A_27a)$

Definition 43 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg (p V0t)))))) \quad (27)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (29)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a. (p (ap V1P V3x))) \vee (p V0Q)))))) \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (33)$$

Assume the following.

$$2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0w \in (ty_2Eordinal_2Eordinal A_27a). (\neg(p (ap (ap (c_2Eordinal_2Eordlt A_27a) V0w) V0w)))) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_27a). (\neg(p (ap (ap (c_2Eordinal_2Eordlt A_27a) V0a) (ap (c_2Eordinal_2EfromNat A_27a) c_2Enum_2E0)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_27a). (\forall V1b \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V0a) (ap (c_2Eordinal_2EordSUC A_27a) V1b))) \Leftrightarrow ((p (ap (c_2Eordinal_2Eordlt A_27a) V0a) V1b)) \vee (V0a = V1b)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}). (\forall V1a \in (ty_2Eordinal_2Eordinal A_27a). (\forall V2b \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2b) (ap (c_2Eordinal_2Esup A_27a) (ap (ap (c_2Epred_set_2EIMAGE (ty_2Eordinal_2Eordinal A_27a) (ty_2Eordinal_2Eordinal A_27a)) V0f) (ap (c_2Eordinal_2Epreds A_27a) V1a)))))) \Leftrightarrow (\exists V3d \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V3d) V1a)) \wedge (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2b) (ap V0f V3d)))))))))) \quad (38)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}), \\
& (((p\ (ap\ V0P\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))) \wedge \\
& ((\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap\ V0P\ V1a)) \Rightarrow \\
& (p\ (ap\ V0P\ (ap\ (c_2Eordinal_2EordSUC\ A_27a)\ V1a)))))) \wedge (\forall V2a \in \\
& (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (c_2Eordinal_2Eomax \\
& A_27a)\ (ap\ (c_2Eordinal_2Epreds\ A_27a)\ V2a)) = (c_2Eoption_2ENONE \\
& (ty_2Eordinal_2Eordinal\ A_27a))) \wedge ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& A_27a)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))\ V2a)) \wedge \\
& (\forall V3b \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& A_27a)\ V3b)\ V2a)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \Rightarrow (p\ (ap\ V0P\ V2a)))))) \Rightarrow (\forall V4a \in \\
& (ty_2Eordinal_2Eordinal\ A_27a).(p\ (ap\ V0P\ V4a))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_27a).(((ap\ (c_2Eordinal_2Eomax\ A_27a)\ (ap\ (c_2Eordinal_2Epreds \\
& A_27a)\ V0a)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal\ A_27a)))) \Leftrightarrow \\
& ((ap\ (c_2Eordinal_2Esup\ A_27a)\ (ap\ (c_2Eordinal_2Epreds\ A_27a \\
& V0a))) = V0a)))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\
& A_27a).(((ap\ (ap\ (c_2Eordinal_2EordADD\ A_27a)\ V0b)\ (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0)) = V0b) \wedge ((\forall V1a \in (ty_2Eordinal_2Eordinal \\
& A_27a).((ap\ (ap\ (c_2Eordinal_2EordADD\ A_27a)\ V0b)\ (ap\ (c_2Eordinal_2EordSUC \\
& A_27a)\ V1a)) = (ap\ (c_2Eordinal_2EordSUC\ A_27a)\ (ap\ (ap\ (c_2Eordinal_2EordADD \\
& A_27a)\ V0b)\ V1a)))))) \wedge (\forall V2a \in (ty_2Eordinal_2Eordinal\ A_27a). \\
& (((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0))\ V2a)) \wedge ((ap\ (c_2Eordinal_2Eomax\ A_27a)\ (\\
& ap\ (c_2Eordinal_2Epreds\ A_27a)\ V2a)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal \\
& A_27a)))))) \Rightarrow ((ap\ (ap\ (c_2Eordinal_2EordADD\ A_27a)\ V0b)\ V2a) = (ap \\
& (c_2Eordinal_2Esup\ A_27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ (ty_2Eordinal_2Eordinal \\
& A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a))\ (ap\ (c_2Eordinal_2EordADD \\
& A_27a)\ V0b))\ (ap\ (c_2Eordinal_2Epreds\ A_27a)\ V2a)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap \\
& (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V1b)\ (ap\ (c_2Eordinal_2Esup\ A_27a) \\
& (ap\ (c_2Eordinal_2Epreds\ A_27a)\ V0a)))))) \Leftrightarrow (\exists V2d \in (ty_2Eordinal_2Eordinal \\
& A_27a).((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V2d)\ V0a)) \wedge (p\ (\\
& ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V1b)\ V2d))))))
\end{aligned} \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (44)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (45)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (46)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (50)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (51)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (52)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (53)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (54)$$

Theorem 1

$$\begin{aligned} & \forall A.27a. \text{nonempty } A.27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ & \quad A.27a). (\forall V1a \in (ty_2Eordinal_2Eordinal A.27a). (\forall V2c \in \\ & \quad (ty_2Eordinal_2Eordinal A.27a). ((p (ap (ap (c.2Eordinal_2Eordlt \\ & \quad A.27a) V1a) V0b)) \Rightarrow (p (ap (ap (c.2Eordinal_2Eordlt A.27a) (ap (ap \\ & \quad (c.2Eordinal_2EordADD A.27a) V2c) V1a)) (ap (ap (c.2Eordinal_2EordADD \\ & \quad A.27a) V2c) V0b)))))))) \end{aligned}$$