thm_2Eordinal_2EordADD__weak__MONO (TMWL16Ajb6qBtzsn8HPtL6EnzA4SMyziKyo)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 4 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (1)$$

Definition 5 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2E$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 9 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 10 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (2)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \quad (3)$$

Definition 11 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{4}$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \tag{5}$$

Definition 12 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{6}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{7}$$

Let $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \tag{8}$$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \tag{9}$$

Definition 13 *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a:\iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{10}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{11}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{12}$$

Definition 14 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 15 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{13}$$

Let $c\_2Epair\_2EFST : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{14}$$

Definition 16 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{15}$$

Definition 17 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 18 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 19 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{16}$$

Definition 20 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

Definition 21 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 22 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 24 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 25 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A.$

Definition 26 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2$

Definition 27 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Definition 28 *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a : \iota.\lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a).$

Definition 29 *We define* c_2Eordinal_2Epreds *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).($

Definition 30 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF)$.

Definition 31 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 32 *We define* c_2Eset__relation_2Emaximal__elements *to be* $\lambda A\_27a : \iota.\lambda V0xs \in (2^{A\_27a}).\lambda V1r \in$

Definition 33 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 34 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 35 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 36 *We define* c_2Eoption_2Esome *to be* $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (ap\ ($c_2Ebool_2ECO

Definition 37 *We define* c_2Eordinal_2Eomax *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap\ ($

Definition 38 *We define* c_2Eordinal_2Eoleast *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap$

Definition 39 *We define* c_2Eordinal_2EordSUC *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{17}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{18}$$

Definition 40 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum}) \tag{19}$$

Let $c\_2Eordinal\_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EordADD\ A\_27a \in (((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \tag{20}$$

Definition 41 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 42 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 43 *We define* c_2Eordinal_2Esup *to be* $\lambda A\_27a : \iota.\lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).($

Definition 44 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 45 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A$

Definition 46 *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A$

Assume the following.

$$True \tag{21}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{22}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{23}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{27}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \tag{33}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{34}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{35}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V1s)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27c\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27c)\ V0f)\ V1s))\ V2t)))))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0x)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1w))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0x)\ V1w))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ord \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V0ord))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))))\tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0b)\ V1a))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V0b)) \lor (V1a = V0b)))))\tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V0a))\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1b))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ V1b)))))\tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2b)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0f)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1a))))) \Leftrightarrow (\exists V3d \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3d)\ V1a)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2b)\ (ap\ V0f\ V3d)))))))))\tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(((p\ (ap\ V0P\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \land ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ V0P\ V1a)) \Rightarrow (p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a))))) \land (\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))) \land ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a)) \land (\forall V3b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3b)\ V2a)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \Rightarrow (p\ (ap\ V0P\ V2a)))))) \Rightarrow (\forall V4a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ V0P\ V4a)))))\tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) = V0b) \wedge ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a)) = (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ V1a)))) \wedge (\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a)) \wedge ((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)))) \Rightarrow ((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ V2a) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b))\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)))))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ V0s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ V0s))\ V1a)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V2b)\ V0s)))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2b)\ V1a)))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s)) \Rightarrow (\forall V2f \in (A\_27b^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V2f\ V0x))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))))))) \tag{46}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{47}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \tag{52}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p))))))))) \tag{53}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{54}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ((p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \tag{55}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p) \Leftrightarrow (\neg(p\ V1q))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{57}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{58}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V2b)) \Rightarrow (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V2b)\ V0c))\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V1a)\ V0c))))))))$$