

thm_2Eordinal_2EordDIV__UNIQUE (TMGzom- nVQHfXvpULoR8eXYrCmMdnujMgPX7)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x)$

Definition 4 We define `c_2Ebool_2EBOUNDED` to be $(\lambda V 0v \in 2. \text{c_2Ebool_2E_2ET}).$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V 0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 6 We define `c_2Emarker_2E_Cong` to be $\lambda V 0x \in 2. V 0x.$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A 0 \ A 1) \tag{2}$$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty_2Epair_2Eprod } A 0 \ A 1) \tag{3}$$

Let `ty_2Ewellorder_2Ewellorder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty_2Ewellorder_2Ewellorder } A 0) \tag{4}$$

Let `c_2Ewellorder_2Ewellorder__REP` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \text{c_2Ewellorder_2Ewellorder__REP } A. 27a \in ((2^{(\text{ty_2Epair_2Eprod } A. 27a \ A. 27a)})(\text{ty_2Ewellorder_2Ewellorder } A. 27a)) \tag{5}$$

Definition 7 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Ebool_2E_7E V0x) V1y))$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (7)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (8)$$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (9)$$

Definition 14 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 15 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40\ 2) V0P))))$

Definition 16 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 17 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 18 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 19 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Ebool_2E_3F V0s) V1t))$

Definition 20 We define $c_Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 21 We define $c_Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a)$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Eordinal_2Eordinal A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS \\ & A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_27a))})}) \end{aligned} \quad (11)$$

Definition 22 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ewellorder_2Ewellorder A_27a)$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS \\ & A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_27a))})^{(ty_2Eordinal_2Eordinal A_27a)}) \end{aligned} \quad (12)$$

Definition 23 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Definition 24 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27b^{A_27a})$

Definition 25 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 26 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder A_27a)^{(2^{(ty_2Epair_2Eprod A_27a A_27a))})} \end{aligned} \quad (13)$$

Definition 27 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 28 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 29 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a)$

Definition 30 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal A_27a)$

Definition 31 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2EIMAGE) P)$

Definition 32 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 33 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (14)$$

Definition 34 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (15)$$

Definition 35 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b))$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (16)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (17)$$

Definition 36 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a))$

Definition 37 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 38 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Epred_set_2EEMPTY\ A_27a\ s))$

Definition 39 We define $c_2Eset_relation_2Emaximal_elements$ to be $\lambda A_27a : \iota. \lambda V0xs \in (2^{A_27a}). \lambda V1r \in (2^{A_27a}). (ap\ (c_2Eset_relation_2EEMPTY\ A_27a\ xs\ r))$

Definition 40 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b\ e))$

Definition 41 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a\ x))$

Definition 42 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2ECOND\ A_27a\ t1\ t2))))$

Definition 43 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a\ P))))$

Definition 44 We define $c_2Eordinal_2Eomax$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Eoption_2ESOME\ A_27a\ s))$

Definition 45 We define $c_2Eordinal_2EordSUC$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a). (ap\ (c_2Eoption_2ESOME\ A_27a\ a))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (18)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (19)$$

Definition 46 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 47 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 48 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (22)$$

Definition 49 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 50 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Eordinal_2EordMOD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordMOD\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (23)$$

Let $c_2Eordinal_2EordDIV : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordDIV\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (24)$$

Let $c_2Eordinal_2EordMULT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordMULT\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (25)$$

Let $c_2Eordinal_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordADD\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (26)$$

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EfromNat\ A_27a \in (ty_2Eordinal_2Eordinal\ A_27a)^{ty_2Enum_2Enum} \quad (27)$$

Definition 51 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 52 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 53 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 54 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 55 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda A_27d : \iota.\lambda V0f$

Definition 56 We define $c_2Equotient_2E_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 57 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27b})$

Definition 58 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x$

Definition 59 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b)$

Definition 60 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).$

Definition 61 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E$

Assume the following.

$$True \quad (28)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.(((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False)))) \quad (31)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (33)$$

Assume the following.

$$((\forall V0t \in 2.(((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))))) \quad (34)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (39)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a.(p\ (ap\ V1Q\ V3x)))))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge (((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V1B) \wedge ((p\ V2C) \vee (p\ V0A))) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (44)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (45)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (46)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (47)$$

Assume the following.

$$(\forall V0v \in 2. ((p (ap c_2Ebool_2EBOUNDED V0v)) \Leftrightarrow True)) \quad (48)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((ap (c_2Ecombin_2EI A_27a) V0x) = V0x)) \quad (49)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT (ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)) (ty_2Eordinal_2Eordinal A_27a)) (c_2Ewellorder_2Eorderiso (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)) (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)) (c_2Eordinal_2Eordinal_ABS A_27a)) (c_2Eordinal_2Eordinal_REP A_27a)))) \quad (50)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal A_27a). (\forall V1y \in (ty_2Eordinal_2Eordinal A_27a). (\forall V2z \in (ty_2Eordinal_2Eordinal A_27a). (((\neg (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V1y) V0x))) \wedge (\neg (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2z) V1y)))))) \Rightarrow (\neg (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2z) V0x)))))) \quad (51)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_27a). (\neg (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V0a) (ap (c_2Eordinal_2EfromNat A_27a) c_2Enum_2E0)))))) \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).((p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ V0a)\ (ap\ (c_2Eordinal_2EordSUC \\ A.27a)\ V1b)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ V0a)\ V1b)) \vee \\ (V0a = V1b)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\ \forall V1m \in ty_2Enum_2Enum.((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ V0n))\ (ap\ (c_2Eordinal_2EfromNat \\ A.27a)\ V1m)))) \Leftrightarrow (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).((ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat \\ A.27a)\ c_2Enum_2E0))\ V0a) = V0a)) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c_2Eordinal_2EordSUC\ A.27a) \\ (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ c_2Enum_2E0)) = (ap\ (c_2Eordinal_2EfromNat \\ A.27a)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\ c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1a \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2c \in \\ (ty_2Eordinal_2Eordinal\ A.27a).(((ap\ (ap\ (c_2Eordinal_2EordADD \\ A.27a)\ V1a)\ V0b) = (ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ V1a)\ V2c)) \Leftrightarrow \\ (V0b = V2c)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).((p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ V0a)\ V1b)) \Leftrightarrow (\exists V2c \in (ty_2Eordinal_2Eordinal \\ A.27a).((\neg(V2c = (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ c_2Enum_2E0))) \wedge \\ (V1b = (ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ V0a)\ V2c)))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1a \in (ty_2Eordinal_2Eordinal\ A.27a).(\neg(p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordADD \\ A.27a)\ V0x)\ V1a))\ V0x)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2c \in \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a).((ap\ (ap\ (c.2Eordinal_2EordADD \\
& \quad A.27a)\ V0a)\ (ap\ (ap\ (c.2Eordinal_2EordADD\ A.27a)\ V1b)\ V2c)) = (ap \\
& \quad (ap\ (c.2Eordinal_2EordADD\ A.27a)\ (ap\ (ap\ (c.2Eordinal_2EordADD \\
& \quad \quad A.27a)\ V0a)\ V1b))\ V2c))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\
& \quad A.27a).(((ap\ (ap\ (c.2Eordinal_2EordMULT\ A.27a)\ V0b)\ (ap\ (c.2Eordinal_2EfromNat \\
& \quad A.27a)\ c.2Enum_2E0)) = (ap\ (c.2Eordinal_2EfromNat\ A.27a)\ c.2Enum_2E0))) \wedge \\
& \quad ((\forall V1a \in (ty_2Eordinal_2Eordinal\ A.27a).((ap\ (ap\ (c.2Eordinal_2EordMULT \\
& \quad A.27a)\ V0b)\ (ap\ (c.2Eordinal_2EordSUC\ A.27a)\ V1a)) = (ap\ (ap\ (c.2Eordinal_2EordADD \\
& \quad A.27a)\ (ap\ (ap\ (c.2Eordinal_2EordMULT\ A.27a)\ V0b)\ V1a))\ V0b))) \wedge \\
& \quad (\forall V2a \in (ty_2Eordinal_2Eordinal\ A.27a).(((p\ (ap\ (ap\ (c.2Eordinal_2Eordlt \\
& \quad A.27a)\ (ap\ (c.2Eordinal_2EfromNat\ A.27a)\ c.2Enum_2E0))\ V2a)) \wedge \\
& \quad ((ap\ (c.2Eordinal_2Eomax\ A.27a)\ (ap\ (c.2Eordinal_2Epreds\ A.27a) \\
& \quad V2a)) = (c.2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal\ A.27a)))))) \Rightarrow \\
& \quad ((ap\ (ap\ (c.2Eordinal_2EordMULT\ A.27a)\ V0b)\ V2a) = (ap\ (c.2Eordinal_2Esup \\
& \quad A.27a)\ (ap\ (ap\ (c.2Epred_set_2EIMAGE\ (ty_2Eordinal_2Eordinal \\
& \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ (c.2Eordinal_2EordMULT \\
& \quad \quad A.27a)\ V0b))\ (ap\ (c.2Eordinal_2Epreds\ A.27a)\ V2a)))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A.27a).(((ap\ (ap\ (c.2Eordinal_2EordMULT\ A.27a)\ V0a)\ (ap\ (c.2Eordinal_2EfromNat \\
& \quad A.27a)\ (ap\ c.2Earithmetic_2ENUMERAL\ (ap\ c.2Earithmetic_2EBIT1 \\
& \quad \quad c.2Earithmetic_2EZERO)))) = V0a)))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2c \in \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a).((\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt \\
& \quad A.27a)\ V1b)\ V0a)))) \Rightarrow (\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ (ap \\
& \quad (ap\ (c.2Eordinal_2EordMULT\ A.27a)\ V2c)\ V1b))\ (ap\ (ap\ (c.2Eordinal_2EordMULT \\
& \quad \quad A.27a)\ V2c)\ V0a)))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2c \in \\ (ty_2Eordinal_2Eordinal\ A.27a).((ap\ (ap\ (c_2Eordinal_2EordMULT \\ A.27a)\ V2c)\ (ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ V0a)\ V1b)) = (ap \\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT \\ A.27a)\ V2c)\ V0a))\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A.27a)\ V2c)\ V1b)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).((p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat\ A.27a) \\ c_2Enum_2E0))\ V1b)) \Rightarrow ((V0a = (ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a) \\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A.27a)\ V1b)\ (ap\ (ap\ (c_2Eordinal_2EordDIV \\ A.27a)\ V0a)\ V1b)))\ (ap\ (ap\ (c_2Eordinal_2EordMOD\ A.27a)\ V0a)\ V1b))) \wedge \\ (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordMOD \\ A.27a)\ V0a)\ V1b))\ V1b)))))) \end{aligned} \quad (65)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\ V0n)\ c_2Enum_2E0)))) \quad (66)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A.27a\ A.27a)\ (c_2Emin_2E_3D\ A.27a))\ (c_2Ecombin_2EI\ A.27a))\ (\\ c_2Ecombin_2EI\ A.27a))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0R1 \in (\\ (2^{A.27a})^{A.27a}).(\forall V1abs1 \in (A.27c^{A.27a}).(\forall V2rep1 \in \\ (A.27a^{A.27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A.27a\ A.27c) \\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A.27b})^{A.27b}).(\forall V4abs2 \in \\ (A.27d^{A.27b}).(\forall V5rep2 \in (A.27b^{A.27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ A.27b\ A.27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ (A.27b^{A.27a})\ (A.27d^{A.27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\ A.27a\ A.27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A.27c \\ A.27b\ A.27a\ A.27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\ A.27a\ A.27d\ A.27c\ A.27b)\ V1abs1)\ V5rep2)))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x \in A_27b.(\forall V4y \in \\
& A_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.(\forall V5y1 \in A_27a.(\forall V6y2 \in A_27a.(((p\ (ap\ (ap\ V0R \\
& V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow \\
& \quad (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))) \\
& \hspace{15em} (71)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2)))))))))) \\
& \hspace{15em} (72)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))) \\
& \hspace{15em} (73)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
& \hspace{15em} (76)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{15em} (77)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (78)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (79)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (80)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (84)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (85)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (86)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (87)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (88)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (89)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (90)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (91)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder A_27a). (p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27a) V0w) V0w))) \quad (92)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder A_27b). ((p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27b) V0w1) V1w2)) \Rightarrow (p (ap (ap (c_2Ewellorder_2Eorderiso A_27b A_27a) V1w2) V0w1)))))) \quad (93)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c.nonempty A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder A_27b). (\forall V2w3 \in (ty_2Ewellorder_2Ewellorder A_27c). (((p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27b) V0w1) V1w2)) \wedge (p (ap (ap (c_2Ewellorder_2Eorderiso A_27b A_27c) V1w2) V2w3))) \Rightarrow (p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27c) V0w1) V2w3)))))) \quad (94)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a). (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder A_27b). ((p (ap (ap (c_2Ewellorder_2Eorderit A_27a A_27b) V0w1) V1w2)) \vee ((p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27b) V0w1) V1w2)) \vee (p (ap (ap (c_2Ewellorder_2Eorderit A_27b A_27a) V1w2) V0w1)))))) \quad (95)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0x0 \in (\\
& \quad ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1y0 \in (ty_2Ewellorder_2Ewellorder \\
& \quad A_27b).(\forall V2a0 \in (ty_2Ewellorder_2Ewellorder\ A_27c).(\\
& \quad \forall V3b0 \in (ty_2Ewellorder_2Ewellorder\ A_27d).(((p\ (ap\ (ap \\
& \quad (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap \\
& \quad (c_2Ewellorder_2Eorderiso\ A_27c\ A_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap \\
& \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap \\
& \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27b\ A_27d)\ V1y0)\ V3b0)))))))))
\end{aligned} \tag{96}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A_27a).(\forall V2q \in \\
& \quad (ty_2Eordinal_2Eordinal\ A_27a).(\forall V3r \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a).(((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\
& \quad A_27a)\ c_2Enum_2E0))\ V1b)) \wedge ((V0a = (ap\ (ap\ (c_2Eordinal_2EordADD \\
& \quad A_27a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V1b)\ V2q))\ V3r)) \wedge \\
& \quad (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V3r)\ V1b)))) \Rightarrow ((ap\ (ap\ (c_2Eordinal_2EordDIV \\
& \quad A_27a)\ V0a)\ V1b) = V2q))))))
\end{aligned}$$