

thm\_2Eordinal\_2EordEXP\_1L  
(TMTswNzhU2dkC9QhJDALW5Sz2hJHVfQv2QS)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_21` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2))) (\lambda V 0x \in 2. V 0x)) (\lambda V 1x \in 2. V 1x)$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } \text{ty\_2Enum\_2Enum} \tag{1}$$

Let `ty_2Esum_2Esum` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \forall A 1. \text{nonempty } A 1 \Rightarrow \text{nonempty } (\text{ty\_2Esum\_2Esum } A 0 A 1) \tag{2}$$

Let `ty_2Ewellorder_2Ewellorder` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty\_2Ewellorder\_2Ewellorder } A 0) \tag{3}$$

Let `ty_2Eordinal_2Eordinal` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A 0. \text{nonempty } A 0 \Rightarrow \text{nonempty } (\text{ty\_2Eordinal\_2Eordinal } A 0) \tag{4}$$

Let `c_2Eordinal_2Eordinal__REP__CLASS` :  $\iota \Rightarrow \iota$  be given. Assume the following.

$$A_{27a} \in ((\text{ty\_2Ewellorder\_2Ewellorder } (\text{ty\_2Esum\_2Esum } \text{ty\_2Enum\_2Enum } A_{27a})) (\text{ty\_2Eordinal\_2Eordinal } A_{27a})) \tag{5}$$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota. (\lambda V 0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A_{27a}})) (\lambda V 1x \in 2. V 1x)) (\lambda V 0x \in 2. V 0x)))$

**Definition 5** We define `c_2Eordinal_2Eordinal__REP` to be  $\lambda A_{27a} : \iota. \lambda V 0a \in (\text{ty\_2Eordinal\_2Eordinal } A_{27a})$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let  $c\_2Ewellorder\_2Ewellorder\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \quad (7)$$

**Definition 6** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 9** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (8)$$

**Definition 10** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ V0x\ V1y))$

**Definition 11** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (10)$$

**Definition 12** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a})$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \quad (11)$$

**Definition 13** We define  $c\_2Eset\_relation\_2Estrict$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

**Definition 14** We define  $c\_Ewellorder\_Eiseg$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_Ewellorder\_Ewellorder\ A\_27a)$

**Definition 15** We define  $c\_Eset\_relation\_Erestrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_Epair\_Eprod\ A\_27a\ A\_27a)})$

Let  $c\_Ewellorder\_Ewellorder\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_Ewellorder\_Ewellorder\_ABS \\ & A\_27a \in ((ty\_Ewellorder\_Ewellorder\ A\_27a)^{(2^{(ty\_Epair\_Eprod\ A\_27a\ A\_27a)})}) \end{aligned} \quad (12)$$

**Definition 16** We define  $c\_Ewellorder\_Ewobound$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1w \in (ty\_Ewellorder\ A\_27a)$

**Definition 17** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_E\_40\ A\_27a)\ P)))$

**Definition 18** We define  $c\_Eset\_relation\_Erange$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)})$

**Definition 19** We define  $c\_Eset\_relation\_Edomain$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_Epair\_Eprod\ A\_27a\ A\_27b)})$

**Definition 20** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_E\_21\ 2)\ (ap\ (c\_Ebool\_E\_21\ 2)\ V1t2))))$

**Definition 21** We define  $c\_Epred\_set\_EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_Epred\_set\_E\_21\ A\_27a)\ s))$

**Definition 22** We define  $c\_Ewellorder\_EelsOf$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_Ewellorder\_Ewellorder\ A\_27a)$

**Definition 23** We define  $c\_Ewellorder\_Eorderiso$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_Ewellorder\_Ewellorder\ A\_27a)$

**Definition 24** We define  $c\_Ewellorder\_Eorderlt$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_Ewellorder\_Ewellorder\ A\_27a)$

**Definition 25** We define  $c\_Eordinal\_Eordlt$  to be  $\lambda A\_27a : \iota. \lambda V0T1 \in (ty\_Eordinal\_Eordinal\ A\_27a)$

**Definition 26** We define  $c\_Eordinal\_Epreds$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_Eordinal\_Eordinal\ A\_27a)$

**Definition 27** We define  $c\_Epred\_set\_EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

**Definition 28** We define  $c\_Epred\_set\_EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap\ (c\_Epred\_set\_E\_21\ A\_27a)\ P))$

**Definition 29** We define  $c\_Eordinal\_Eoleast$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(ty\_Eordinal\_Eordinal\ A\_27a)})$

**Definition 30** We define  $c\_Eordinal\_Eesup$  to be  $\lambda A\_27a : \iota. \lambda V0ordset \in (2^{(ty\_Eordinal\_Eordinal\ A\_27a)})$

Let  $ty\_Eone\_Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_Eone\_Eone \quad (13)$$

**Definition 31** We define  $c\_Eone\_Eone$  to be  $(ap\ (c\_Emin\_E\_40\ ty\_Eone\_Eone)\ (\lambda V0x \in ty\_Eone\_Eone))$

Let  $c\_Esum\_EABS\_sum : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_Esum\_EABS\_sum \\ & A\_27a\ A\_27b \in ((ty\_Esum\_Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (14)$$

**Definition 32** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (15)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (16)$$

**Definition 33** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a))$

**Definition 34** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 35** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2EF$

**Definition 36** We define  $c\_2Eset\_relation\_2Emaximal\_elements$  to be  $\lambda A\_27a : \iota. \lambda V0xs \in (2^{A\_27a}). \lambda V1r \in$

**Definition 37** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_Esum\_2EABS$

**Definition 38** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_ABS$

**Definition 39** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (c\_2Ebool\_2EF$

**Definition 40** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{A\_27a}). (ap (ap (ap (c\_2Ebool\_2ECOND$

**Definition 41** We define  $c\_2Eordinal\_2Eomax$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{(ty\_2Eordinal\_2Eordinal A\_27a)}). (ap (c\_2Eordinal\_2Eordinal$

Let  $c\_2Eordinal\_2EordMULT : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eordinal\_2EordMULT A\_27a \in ((ty\_2Eordinal\_2Eordinal A\_27a)^{(ty\_2Eordinal\_2Eordinal A\_27a)}) \quad (17)$$

**Definition 42** We define  $c\_2Eordinal\_2EordSUC$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Eordinal\_2Eordinal A\_27a)$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (18)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (19)$$

**Definition 43** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

**Definition 44** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (21)$$

**Definition 45** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (22)$$

**Definition 46** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 47** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a \in ( (ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum} ) \quad (23)$$

Let  $c\_2Eordinal\_2EordEXP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EordEXP\ A\_27a \in ( ( (ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} )^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} ) \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A\_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& (p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in 2.(\forall V1P\_27 \in 2.(\forall V2Q \in 2.(\forall V3Q\_27 \in \\
& 2.(((p \ V2Q) \Rightarrow ((p \ V0P) \Leftrightarrow (p \ V1P\_27))) \wedge ((p \ V1P\_27) \Rightarrow ((p \ V2Q) \Leftrightarrow (p \ V3Q\_27)))) \Rightarrow \\
& (((p \ V0P) \wedge (p \ V2Q)) \Leftrightarrow ((p \ V1P\_27) \wedge (p \ V3Q\_27))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal \\ A.27a).(\forall V1w \in (ty\_2Eordinal\_2Eordinal\ A.27a).((p\ (ap \\ (ap\ (c\_2Ebool\_2EIN\ (ty\_2Eordinal\_2Eordinal\ A.27a))\ V0x)\ (ap\ ( \\ c\_2Eordinal\_2Epreds\ A.27a)\ V1w))) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt \\ A.27a)\ V0x)\ V1w)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal \\ A.27a).((ap\ (c\_2Eordinal\_2Esup\ A.27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT \\ (ty\_2Eordinal\_2Eordinal\ A.27a))\ V0a)\ (c\_2Epred\_set\_2EEMPTY \\ (ty\_2Eordinal\_2Eordinal\ A.27a)))))) = V0a)) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A.27a)}). \\ (((p\ (ap\ V0P\ (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ c\_2Enum\_2E0)))) \wedge \\ ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A.27a).((p\ (ap\ V0P\ V1a)) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (c\_2Eordinal\_2EordSUC\ A.27a)\ V1a)))))) \wedge (\forall V2a \in \\ (ty\_2Eordinal\_2Eordinal\ A.27a).(((ap\ (c\_2Eordinal\_2Eomax \\ A.27a)\ (ap\ (c\_2Eordinal\_2Epreds\ A.27a)\ V2a)) = (c\_2Eoption\_2ENONE \\ (ty\_2Eordinal\_2Eordinal\ A.27a)))) \wedge ((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt \\ A.27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ c\_2Enum\_2E0))\ V2a)) \wedge \\ (\forall V3b \in (ty\_2Eordinal\_2Eordinal\ A.27a).((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt \\ A.27a)\ V3b)\ V2a)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \Rightarrow (p\ (ap\ V0P\ V2a)))))) \Rightarrow (\forall V4a \in \\ (ty\_2Eordinal\_2Eordinal\ A.27a).(p\ (ap\ V0P\ V4a)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in ty\_2Enum\_2Enum.( \\ \forall V1y \in ty\_2Enum\_2Enum.(((ap\ (c\_2Eordinal\_2EfromNat\ A.27a) \\ V0x) = (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal \\ A.27a).((ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A.27a)\ V0a)\ (ap\ (c\_2Eordinal\_2EfromNat \\ A.27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))))) = V0a)) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in (ty\_2Eordinal\_2Eordinal \\
& A\_27a).(ap\ (ap\ (c\_2Eordinal\_2EordEXP\ A\_27a)\ V0a)\ (ap\ (c\_2Eordinal\_2EfromNat \\
& A\_27a)\ c\_2Enum\_2E0)) = (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V1a \in \\
& (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2a\_27 \in (ty\_2Eordinal\_2Eordinal \\
& A\_27a).(ap\ (ap\ (c\_2Eordinal\_2EordEXP\ A\_27a)\ V1a)\ (ap\ (c\_2Eordinal\_2EordSUC \\
& A\_27a)\ V2a\_27)) = (ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ (ap\ (ap \\
& (c\_2Eordinal\_2EordEXP\ A\_27a)\ V1a)\ V2a\_27))\ V1a)))) \wedge ((\forall V3a \in \\
& (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V4a\_27 \in (ty\_2Eordinal\_2Eordinal \\
& A\_27a).((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat \\
& A\_27a)\ c\_2Enum\_2E0))\ V4a\_27)) \wedge ((ap\ (c\_2Eordinal\_2Eomax\ A\_27a) \\
& (ap\ (c\_2Eordinal\_2Epreds\ A\_27a)\ V4a\_27)) = (c\_2Eoption\_2ENONE \\
& (ty\_2Eordinal\_2Eordinal\ A\_27a)))) \Rightarrow ((ap\ (ap\ (c\_2Eordinal\_2EordEXP \\
& A\_27a)\ V3a)\ V4a\_27) = (ap\ (c\_2Eordinal\_2Esup\ A\_27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a)) \\
& (ap\ (c\_2Eordinal\_2EordEXP\ A\_27a)\ V3a))\ (ap\ (c\_2Eordinal\_2Epreds \\
& A\_27a)\ V4a\_27)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in \\
& (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V2x)\ V1t))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap \\
& (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A\_27a))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\
& A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a) \\
& V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A\_27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\
& V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V0x)\ V2s))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). \\
& ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\
& A\_27a\ A\_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x)) \wedge \\
& (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V3x)\ V1s))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{50}$$



Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (64)$$

**Theorem 1**

$$\forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow (\forall V0a \in (\text{ty\_2Eordinal\_2Eordinal } A_{.27a}). ((\text{ap } (\text{ap } (\text{c\_2Eordinal\_2EordEXP } A_{.27a}) (\text{ap } (\text{c\_2Eordinal\_2EfromNat } A_{.27a}) (\text{ap } \text{c\_2Earithmetic\_2ENUMERAL } (\text{ap } \text{c\_2Earithmetic\_2EBIT1 } \text{c\_2Earithmetic\_2EZERO)))))) V0a) = (\text{ap } (\text{c\_2Eordinal\_2EfromNat } A_{.27a}) (\text{ap } \text{c\_2Earithmetic\_2ENUMERAL } (\text{ap } \text{c\_2Earithmetic\_2EBIT1 } \text{c\_2Earithmetic\_2EZERO))))))$$