

thm_2Eordinal_2EordMULT__ASSOC (TMFRz32opTj2oXGoGg8XoeG1BAHtUHAeeir)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^{A}.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 8 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 10 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 11 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (6)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ewellorder_2Ewellorder A0) \quad (7)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eordinal_2Eordinal A0) \quad (8)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Eenum_2Eenum A_27a))})^{(ty_2Eordinal_2Eordinal A_27a)}) \quad (9)$$

Definition 12 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (10)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)}) \quad (11)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (12)$$

Definition 13 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod$

Definition 14 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 15 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (15)$$

Definition 16 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 17 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 18 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \end{aligned} \quad (16)$$

Definition 19 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder$

Definition 20 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 21 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A$

Definition 22 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod$

Definition 23 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 24 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c$

Definition 25 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A$

Definition 26 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 27 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 28 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a).\lambda$

Definition 29 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal A_27a).($

Definition 30 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ETF).$

Definition 31 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2$

Definition 32 We define $c_2Eset_relation_2Emaximal_elements$ to be $\lambda A_27a : \iota.\lambda V0xs \in (2^{A_27a}).\lambda V1r \in$

Definition 33 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Definition 34 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2Eoption_2E$

Definition 35 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 36 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap (ap (ap (c_2Ebool_2ECOND$

Definition 37 We define $c_2Eordinal_2Eomax$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).(ap ($

Definition 38 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).(ap ($

Definition 39 We define $c_2Eordinal_2EordSUC$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{17}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{18}$$

Definition 40 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP).$

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2EfromNat A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{ty_2Enum_2Enum}) \tag{19}$$

Definition 41 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 42 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E$

Definition 43 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).($

Definition 44 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ETF).$

Definition 45 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A$

Definition 46 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A$

Let $c_2Eordinal_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordADD\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (20)$$

Let $c_2Eordinal_2EordMULT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordMULT\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \quad (21)$$

Assume the following.

$$True \quad (22)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\
& 2^{A_27a}). ((\exists V2x \in A_27a. ((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\
& V0P) \wedge (\exists V3x \in A_27a. (p (ap V1Q V3x))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\
& (p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\
& 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow \\
& ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in 2. (\forall V1P_27 \in 2. (\forall V2Q \in 2. (\forall V3Q_27 \in \\
& 2. (((p V2Q) \Rightarrow ((p V0P) \Leftrightarrow (p V1P_27))) \wedge ((p V1P_27) \Rightarrow ((p V2Q) \Leftrightarrow (p V3Q_27)))) \Rightarrow \\
& (((p V0P) \wedge (p V2Q)) \Leftrightarrow ((p V1P_27) \wedge (p V3Q_27))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1a \in \\
& A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (\\
& ap V0P V1a))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow \forall A_27c. \\
& nonempty A_27c \Rightarrow (\forall V0f \in (A_27c^{A_27a}). (\forall V1s \in (2^{A_27a}). \\
& (\forall V2t \in (2^{A_27b}). ((p (ap (ap (c_2Ecardinal_2Ecardleq A_27a \\
& A_27b) V1s) V2t)) \Rightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq A_27c A_27b) \\
& (ap (ap (c_2Epred_set_2EIMAGE A_27a A_27c) V0f) V1s)) V2t))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\
& A_27a). (\forall V1w \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap \\
& (ap (c_2Ebool_2EIN (ty_2Eordinal_2Eordinal A_27a) V0x) (ap (\\
& c_2Eordinal_2Epreds A_27a) V1w))) \Leftrightarrow (p (ap (ap (c_2Eordinal_2Eordlt \\
& A_27a) V0x) V1w))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ord \in (ty_2Eordinal_2Eordinal \\ A_27a).(p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal \\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ (ap\ (c_2Eordinal_2Epreds \\ A_27a)\ V0ord))\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\ A_27a)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\ ((p\ (ap\ V0P\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))) \wedge \\ ((\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap\ V0P\ V1a)) \Rightarrow \\ (p\ (ap\ V0P\ (ap\ (c_2Eordinal_2EordSUC\ A_27a)\ V1a)))))) \wedge (\forall V2a \in \\ (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (c_2Eordinal_2Eomax \\ A_27a)\ (ap\ (c_2Eordinal_2Epreds\ A_27a)\ V2a)) = (c_2Eoption_2ENONE \\ (ty_2Eordinal_2Eordinal\ A_27a))) \wedge ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))\ V2a)) \wedge \\ (\forall V3b \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A_27a)\ V3b)\ V2a)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \Rightarrow (p\ (ap\ V0P\ V2a)))))) \Rightarrow (\forall V4a \in \\ (ty_2Eordinal_2Eordinal\ A_27a).(p\ (ap\ V0P\ V4a)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in ty_2Enum_2Enum.(\\ \forall V1y \in ty_2Enum_2Enum.(((ap\ (c_2Eordinal_2EfromNat\ A_27a) \\ V0x) = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a).(\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a).(\forall V2c \in \\ (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (ap\ (c_2Eordinal_2EordADD \\ A_27a)\ V1a)\ V0b) = (ap\ (ap\ (c_2Eordinal_2EordADD\ A_27a)\ V1a)\ V2c)) \Leftrightarrow \\ (V0b = V2c)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\
& A_27a).(((ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V0b)\ (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0)) = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0)) \wedge \\
& ((\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a).((ap\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V0b)\ (ap\ (c_2Eordinal_2EordSUC\ A_27a)\ V1a)) = (ap\ (ap\ (c_2Eordinal_2EordADD \\
& A_27a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V0b)\ V1a))\ V0b)))) \wedge \\
& (\forall V2a \in (ty_2Eordinal_2Eordinal\ A_27a).(((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& A_27a)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))\ V2a)) \wedge \\
& ((ap\ (c_2Eordinal_2Eomax\ A_27a)\ (ap\ (c_2Eordinal_2Epreds\ A_27a)\ \\
& V2a)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal\ A_27a)))) \Rightarrow \\
& ((ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V0b)\ V2a) = (ap\ (c_2Eordinal_2Esup \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ (ty_2Eordinal_2Eordinal \\
& A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a))\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V0b))\ (ap\ (c_2Eordinal_2Epreds\ A_27a)\ V2a))))))))) \\
& \hspace{10em} (42)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0z \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1x \in (ty_2Eordinal_2Eordinal\ A_27a).(\forall V2y \in \\
& (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V0z)\ V1x) = (ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V0z)\ V2y))) \Leftrightarrow \\
& ((V0z = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0)) \vee (V1x = \\
& V2y)))))) \\
& \hspace{10em} (43)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).((\\
& p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal\ A_27a) \\
& (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V1s)\ (c_2Epred_set_2EUNIV \\
& (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \Rightarrow ((ap\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V0a)\ (ap\ (c_2Eordinal_2Esup\ A_27a)\ V1s)) = (ap\ (c_2Eordinal_2Esup \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ (ty_2Eordinal_2Eordinal \\
& A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a))\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V0a))\ V1s)))))) \\
& \hspace{10em} (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A_27a).(\forall V2c \in \\
& (ty_2Eordinal_2Eordinal\ A_27a).((ap\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V2c)\ (ap\ (ap\ (c_2Eordinal_2EordADD\ A_27a)\ V0a)\ V1b)) = (ap \\
& (ap\ (c_2Eordinal_2EordADD\ A_27a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT \\
& A_27a)\ V2c)\ V0a))\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V2c)\ V1b)))))) \\
& \hspace{10em} (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1t \in \\ & (2^{A_27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & A_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2x)\ V1t))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))) \end{aligned} \quad (47)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ & A_27a). (\forall V1b \in (ty_2Eordinal_2Eordinal\ A_27a). (\forall V2c \in \\ & (ty_2Eordinal_2Eordinal\ A_27a). ((ap\ (ap\ (c_2Eordinal_2EordMULT \\ & A_27a)\ V0a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V1b)\ V2c)) = (\\ & ap\ (ap\ (c_2Eordinal_2EordMULT\ A_27a)\ (ap\ (ap\ (c_2Eordinal_2EordMULT \\ & A_27a)\ V0a)\ V1b))\ V2c)))) \end{aligned}$$