thm_2Eordinal_2EordMULT__continuous (TMFF5A6vXjjENbc218ENyRr78Z9s7jT2xcN)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Definition 8 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 13 *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0s1\in(2^{A\_27a}).\lambda V1s2\in(2^{A$

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t\in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $ty\_2Ewellorder\_2Ewellorder:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \quad (3)$$

Let $ty\_2Eordinal\_2Eordinal:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \quad (4)$$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a\in((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \quad (5)$$

Definition 15 *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a:\iota.\lambda V0a\in(ty\_2Eordinal\_2Eordinal\ A$

Let $ty\_2Epair\_2Eprod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0\Rightarrow\forall A1.nonempty\ A1\Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (6)$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a\in((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \quad (7)$$

Let $c\_2Epair\_2EABS\_\_prod:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b\in((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (8)$$

Definition 16 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0x\in A\_27a.\lambda V1y\in A\_27b.(ap\ (c\_2$

Let $c\_2Epair\_2ESND:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b\in(A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (9)$$

Let $c\_2Epair\_2EFST:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b\in(A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (10)$$

Definition 17 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda A\_27c:\iota.\lambda V0f\in((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC:\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b\in((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 18 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 19 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 20 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS:\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a\in((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{12}$$

Definition 21 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a:\iota.\lambda V0x\in A\_27a.\lambda V1w\in(ty\_2Ewellord$

Definition 22 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod\ A}$

Definition 23 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r\in(2^{(ty\_2Epair\_2Eprod}$

Definition 24 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ (c.$

Definition 25 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Ewellorder\_2Ewellorder\ A.$

Definition 26 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1\in(ty\_2Ewellorder\_2I$

Definition 27 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1\in(ty\_2Ewellorder\_2E$

Definition 28 *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a:\iota.\lambda V0T1\in(ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

Definition 29 *We define* c_2Eordinal_2Epreds *to be* $\lambda A\_27a:\iota.\lambda V0w\in(ty\_2Eordinal\_2Eordinal\ A\_27a).($

Definition 30 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0f\in(A\_27b^{A\_27a}).\lambda V1s\in$

Definition 31 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 32 *We define* c_2Eordinal_2Eoleast *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap$

Definition 33 *We define* c_2Eordinal_2Esup *to be* $\lambda A\_27a:\iota.\lambda V0ordset\in(2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).($

Let $ty\_2Eone\_2Eone:\iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{13}$$

Definition 34 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x\in ty\_2Eone\_2$

Let $c\_2Esum\_2EABS\_\_sum : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (14)$$

Definition 35 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (15)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (16)$$

Definition 36 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 37 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 38 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 39 *We define* c_2Eset__relation_2Emaximal__elements *to be* $\lambda A\_27a : \iota.\lambda V0xs \in (2^{A\_27a}).\lambda V1r \in$

Definition 40 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 41 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 42 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 43 *We define* c_2Eoption_2Esome *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EC}$

Definition 44 *We define* c_2Eordinal_2Eomax *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap\ ($

Let $c\_2Eordinal\_2EordADD : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eordinal\_2EordADD\ A\_27a \in (((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \quad (17)$$

Definition 45 *We define* c_2Eordinal_2EordSUC *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (18)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (19)$$

Definition 46 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Eordinal\_2EordMULT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EordMULT\ A\_27a \in (((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \tag{21}$$

Assume the following.

$$True \tag{22}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{24}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg (p\ V0t)))) \tag{25}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{28}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{32}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{33}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))) = (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in ty\_2Enum\_2Enum.(\forall V1y \in ty\_2Enum\_2Enum.(((ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ V0x) = (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0f \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V1a))\wedge((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))))\Rightarrow((ap\ V0f\ V1a) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0f)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1a))))))\wedge(\forall V2x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V3y \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3y)\ V2x)))\Rightarrow(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ V0f\ V3y))\ (ap\ V0f\ V2x)))))))\Rightarrow(\forall V4s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ V4s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))))\wedge(\neg(V4s = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)))))\Rightarrow((ap\ V0f\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ V4s)) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0f)\ V4s)))))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0b)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) = (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\wedge((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0b)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a)) = (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0b)\ V1a))\ V0b)))\wedge(\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a))\wedge((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))))\Rightarrow((ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0b)\ V2a) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0b))\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a))))))))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0c)\ V1a))\ (ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0c)\ V2b))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V2b)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V0c)))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V0f)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a)) = (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27b))) \tag{40}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ V1s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))) \Rightarrow ((ap\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0a)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ V1s)) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (c\_2Eordinal\_2EordMULT\ A\_27a)\ V0a))\ V1s))))))$$