

thm_2Eordinal_2Eorderiso__wobound2
(TMJa8AfjoXpHLTeuF4UBLAccUYK9iC9wKUs)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (1)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ewellorder_2Ewellorder A0) \quad (2)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A.\lambda 27a \in ((2^{(ty_2Epair_2Eprod A.\lambda 27a A.\lambda 27a)})(ty_2Ewellorder_2Ewellorder A.\lambda 27a)) \quad (3)$$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.\lambda 27a.nonempty A.\lambda 27a \Rightarrow \forall A.\lambda 27b.nonempty A.\lambda 27b \Rightarrow c_2Epair_2EABS_prod A.\lambda 27a A.\lambda 27b \in ((ty_2Epair_2Eprod A.\lambda 27a A.\lambda 27b)^{(2^{A-27b})^{A-27a}}) \quad (4)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (5)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (6)$$

Definition 11 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a}$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow \forall A_27b. nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2^{A_27b})}) \end{aligned} \quad (7)$$

Definition 12 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 13 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 14 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (8)$$

Definition 15 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellord$

Definition 16 We define c_2Emin_2E40 to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap\ P\ x)) \text{ then } (the (\lambda x. x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 17 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E40$

Definition 18 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A$

Definition 19 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod$

Definition 20 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c$

Definition 21 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A$

Definition 22 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 23 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Assume the following.

$$True \quad (9)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (12)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder A_27a).(\forall V1w2 \in (ty_2Ewellorder_2Ewellorder A_27b).((p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27b) V0w1) V1w2)) \Rightarrow (\neg(p (ap (ap (c_2Ewellorder_2Eorderlt A_27a A_27b) V0w1) V1w2)))))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y) = (ap (ap (c_2Epair_2E_2C A_27a A_27b) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (16)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A.27a\ 2)^{A.27b}).(\forall V1v \in \\ & A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A.27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A.27a\ A.27b)\ V0f))) \Leftrightarrow (\exists V2x \in A.27b.((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A.27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Rightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (p\ V1q)) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge ((\\ & \quad \neg(p\ V1q)) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0a \in A.27a.(\forall V1b \in \\ A.27a.(\forall V2w \in (ty_2Ewellorder_2Ewellorder A.27a).((p \\ (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) (ap (ap \\ (c_2Epair_2E_2C A.27a A.27a) V0a) V1b)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V2w)))) \Rightarrow (\\ (ap (ap (c_2Ewellorder_2Ewobound A.27a) V0a) (ap (ap (c_2Ewellorder_2Ewobound \\ A.27a) V1b) V2w)) = (ap (ap (c_2Ewellorder_2Ewobound A.27a) V0a) \\ V2w)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1w \in \\ (ty_2Ewellorder_2Ewellorder A.27a).((ap (c_2Ewellorder_2EelsOf \\ A.27a) (ap (ap (c_2Ewellorder_2Ewobound A.27a) V0x) V1w)) = (ap \\ (c_2Epred_set_2EGSPEC A.27a A.27a) (\lambda V2y \in A.27a.(ap (ap (\\ c_2Epair_2E_2C A.27a 2) V2y) (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod \\ A.27a A.27a)) (ap (ap (c_2Epair_2E_2C A.27a A.27a) V2y) V0x)) (ap \\ (c_2Eset_relation_2Estrict A.27a) (ap (c_2Ewellorder_2Ewellorder_REP \\ A.27a) V1w))))))))) \end{aligned} \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ A.27a).(p (ap (ap (c_2Ewellorder_2Eorderiso A.27a A.27a) V0w) \\ V0w))) \quad (31)$$

Theorem 1

$$\begin{aligned} \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1w \in \\ (ty_2Ewellorder_2Ewellorder A.27a).(\forall V2y \in A.27a.((p \\ (ap (ap (c_2Ewellorder_2Eorderiso A.27a A.27a) (ap (ap (c_2Ewellorder_2Ewobound \\ A.27a) V0x) V1w)) (ap (ap (c_2Ewellorder_2Ewobound A.27a) V2y) \\ V1w)))) \Rightarrow (\neg(p (ap (ap (c_2Ebool_2EIN (ty_2Epair_2Eprod A.27a A.27a)) \\ (ap (ap (c_2Epair_2E_2C A.27a A.27a) V0x) V2y)) (ap (c_2Eset_relation_2Estrict \\ A.27a) (ap (c_2Ewellorder_2Ewellorder_REP A.27a) V1w))))))))) \end{aligned}$$