

thm_2Eordinal_2Eordinal_IVT (TMHD2E54woLusPStU7rPB1Q67nFr2idwqut)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.^{27a} : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Emarker_2E_Abbrev$ to be $\lambda V0x \in 2.V0x$.

Definition 6 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF))$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $ty_2Esum_2E_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2E_sum\ A0\ A1) \tag{2}$$

Let $ty_2Ewellorder_2E_wellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2E_wellorder\ A0) \tag{3}$$

Let $ty_2Eordinal_2E_ordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2E_ordinal\ A0) \tag{4}$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_27a))})^{(ty_2Eordinal_2Eordinal A_27a)})^{\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS} \quad (5)$$

Definition 9 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)})^{\forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP} \quad (7)$$

Definition 10 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E21 2) (\lambda V2t \in 2. (ap V1f V0x))))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 11 We define c_2Epair_2E2C to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Ebool_2E21 2) (ap V1f V0x))$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a)^{(ty_2Epair_2Eprod A_27a A_27b)} \quad (10)$$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (11)$$

Definition 14 We define $c_Eset_relation_Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$,

Definition 15 We define $c_Ewellorder_Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 16 We define $c_Eset_relation_Erestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (12)$$

Definition 17 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder\ A_27a)$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P)))$

Definition 19 We define $c_Eset_relation_ERange$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 20 We define $c_Eset_relation_EDomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t1)\ t2)))$

Definition 22 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Emin_2E_40\ A_27a)\ s)\ t)$

Definition 23 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 24 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 25 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 26 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a). \lambda V1T2 \in (ty_2Eordinal_2Eordinal\ A_27a). (ap\ (c_2Emin_2E_40\ A_27a)\ T1)\ T2)$

Definition 27 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Emin_2E_40\ A_27a)\ P)$

Definition 28 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Emin_2E_40\ A_27a)\ s)\ f)$

Definition 29 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a). (ap\ (c_2Emin_2E_40\ A_27a)\ w)$

Definition 30 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2EUNION\ A_27a)\ P)$

Definition 31 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota. \lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Emin_2E_40\ A_27a)\ ordset)$

Definition 32 We define $c_2Eordinal_2EordSUC$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a). (ap\ (c_2Emin_2E_40\ A_27a)\ a)$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (13)$$

Definition 33 We define c_2Eone_2Eone to be $(ap\ (c_2Emin_2E_40\ ty_2Eone_2Eone)\ (\lambda V0x \in ty_2Eone_2Eone. x))$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum \\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \end{aligned} \quad (14)$$

Definition 34 We define c_2Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eoption_2Eoption\ A0) \quad (15)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eoption_2Eoption_ABS\ A_27a \in \\ ((ty_2Eoption_2Eoption\ A_27a)^{(ty_2Esum_2Esum\ A_27a\ ty_2Eone_2Eone)}) \end{aligned} \quad (16)$$

Definition 35 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ 0)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (17)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (18)$$

Definition 36 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EfromNat\ A_27a \in (\\ (ty_2Eordinal_2Eordinal\ A_27a)^{ty_2Enum_2Enum}) \end{aligned} \quad (19)$$

Definition 37 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Definition 38 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2EF\ V0x)\ V1s)$

Definition 39 We define $c_2Eset_relation_2Emaximal_elements$ to be $\lambda A_27a : \iota. \lambda V0xs \in (2^{A_27a}). \lambda V1r \in (2^{A_27a}). (ap\ (c_2Ebool_2EF\ V0xs)\ V1r)$

Definition 40 We define c_2Esum_2EINL to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27a. (ap\ (c_2Esum_2EABS_sum\ A_27a\ A_27b)\ V0e)$

Definition 41 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. (ap\ (c_2Eoption_2Eoption_ABS\ A_27a)\ V0x)$

Definition 42 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. (c_2Ebool_2EF\ V0t1\ V2t2)\ V1t1))\ V0t)$

Definition 43 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{A_27a}). (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V0x)\ V0x)$

Definition 44 We define $c_2Eordinal_2Eomax$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ V0s)$

Assume the following.

$$True \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (26)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (28)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (29)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in \\ 2. (((\forall V2x \in A_27a. (p\ (ap\ V0P\ V2x))) \wedge (p\ V1Q))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ p\ V0A) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge (\neg(p\ V1B)))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (\\ (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a). (\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a). ((\neg(p\ (\\ ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V0b)\ V1a))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A_27a)\ V1a)\ V0b)) \vee (V1a = V0b)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a). (\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a). (((\neg(p\ (\\ ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V0b)\ V1a))) \wedge (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A_27a)\ V1a)\ V0b)))) \Rightarrow (V1a = V0b)))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1y \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2z \in \\ (ty_2Eordinal_2Eordinal\ A.27a).(((\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt \\ A.27a)\ V1y)\ V0x)))) \wedge (\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V2z) \\ V1y)))))) \Rightarrow (\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V2z)\ V0x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1y \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V2z \in \\ (ty_2Eordinal_2Eordinal\ A.27a).(((p\ (ap\ (ap\ (c.2Eordinal_2Eordlt \\ A.27a)\ V0x)\ V1y)) \wedge (\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V2z) \\ V1y)))))) \Rightarrow (p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V0x)\ V2z)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0Q \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}). \\ (\forall V1P \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}).(((\exists V2a \in \\ (ty_2Eordinal_2Eordinal\ A.27a).(p\ (ap\ V1P\ V2a))) \wedge (\forall V3a \in \\ (ty_2Eordinal_2Eordinal\ A.27a).(((\forall V4b \in (ty_2Eordinal_2Eordinal_2Eordinal \\ A.27a).((p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V4b)\ V3a)) \Rightarrow (\neg \\ p\ (ap\ V1P\ V4b)))))) \wedge (p\ (ap\ V1P\ V3a)) \Rightarrow (p\ (ap\ V0Q\ V3a)))))) \Rightarrow (p\ (ap\ V0Q \\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V1P)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\neg(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V0a)\ (ap\ (c.2Eordinal_2Eordlt \\ A.27a)\ c.2Enum_2E0)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V0a)\ (ap\ (c.2Eordinal_2EordSUC \\ A.27a)\ V0a)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\ A.27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A.27a).((p\ (ap \\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V0a)\ (ap\ (c.2Eordinal_2EordSUC \\ A.27a)\ V1b))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Eordinal_2Eordlt\ A.27a)\ V0a)\ V1b)) \vee \\ (V0a = V1b)))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal \\
& A_{.27a})(ty_2Eordinal_2Eordinal\ A_{.27a})).(\forall V1a \in (ty_2Eordinal_2Eordinal \\
& A_{.27a})).(\forall V2b \in (ty_2Eordinal_2Eordinal\ A_{.27a})).((p\ (ap \\
& (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ V2b)\ (ap\ (c_2Eordinal_2Esup\ A_{.27a}) \\
& (ap\ (ap\ (c_2Epred_set_2EIMAGE\ (ty_2Eordinal_2Eordinal\ A_{.27a}) \\
& (ty_2Eordinal_2Eordinal\ A_{.27a}))\ V0f)\ (ap\ (c_2Eordinal_2Epreds \\
& A_{.27a})\ V1a)))))) \Leftrightarrow (\exists V3d \in (ty_2Eordinal_2Eordinal\ A_{.27a}). \\
& ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ V3d)\ V1a)) \wedge (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& A_{.27a})\ V2b)\ (ap\ V0f\ V3d)))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\
& A_{.27a}).((\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ (ap\ (c_2Eordinal_2EfromNat \\
& A_{.27a})\ c_2Enum_2E0))\ V0x)))) \Leftrightarrow (V0x = (ap\ (c_2Eordinal_2EfromNat \\
& A_{.27a})\ c_2Enum_2E0)))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_{.27a}).(((ap\ (c_2Eordinal_2Eomax\ A_{.27a})\ (ap\ (c_2Eordinal_2Epreds \\
& A_{.27a})\ V0a)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal\ A_{.27a}))) \Leftrightarrow \\
& ((ap\ (c_2Eordinal_2Esup\ A_{.27a})\ (ap\ (c_2Eordinal_2Epreds\ A_{.27a}) \\
& V0a)) = V0a)))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_{.27a}).((V0a = (ap\ (c_2Eordinal_2EfromNat\ A_{.27a})\ c_2Enum_2E0)) \vee \\
& ((\exists V1a0 \in (ty_2Eordinal_2Eordinal\ A_{.27a}).(V0a = (ap\ (c_2Eordinal_2EordSUC \\
& A_{.27a})\ V1a0)))) \vee ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ (ap\ (c_2Eordinal_2EfromNat \\
& A_{.27a})\ c_2Enum_2E0))\ V0a)) \wedge ((ap\ (c_2Eordinal_2Eomax\ A_{.27a})\ (\\
& ap\ (c_2Eordinal_2Epreds\ A_{.27a})\ V0a)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal \\
& A_{.27a})))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((ap\ (c_2Eordinal_2Eomax\ A_{.27a}) \\
& (ap\ (c_2Eordinal_2Epreds\ A_{.27a})\ (ap\ (c_2Eordinal_2EfromNat\ A_{.27a}) \\
& c_2Enum_2E0))) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal \\
& A_{.27a})))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (51)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (54)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (59)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (62)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (63)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (64)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal \\ & A_27a)(ty_2Eordinal_2Eordinal \ A_27a)).(\forall V1a1 \in (ty_2Eordinal_2Eordinal \\ & A_27a).(\forall V2a2 \in (ty_2Eordinal_2Eordinal \ A_27a).(\forall V3c \in \\ & (ty_2Eordinal_2Eordinal \ A_27a).((\forall V4a \in (ty_2Eordinal_2Eordinal \\ & A_27a).((p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ (ap \ (c_2Eordinal_2EfromNat \\ & A_27a) \ c_2Enum_2E0)) \ V4a)) \wedge ((ap \ (c_2Eordinal_2Eomax \ A_27a) \ (\\ & ap \ (c_2Eordinal_2Epreds \ A_27a) \ V4a)) = (c_2Eoption_2ENONE \ (ty_2Eordinal_2Eordinal \\ & A_27a)))))) \Rightarrow ((ap \ V0f \ V4a) = (ap \ (c_2Eordinal_2Esup \ A_27a) \ (ap \ (ap \\ & (c_2Epred_set_2EIMAGE \ (ty_2Eordinal_2Eordinal \ A_27a) \ (ty_2Eordinal_2Eordinal \\ & A_27a)) \ V0f) \ (ap \ (c_2Eordinal_2Epreds \ A_27a) \ V4a)))))) \wedge ((\forall V5x \in \\ & (ty_2Eordinal_2Eordinal \ A_27a).(\forall V6y \in (ty_2Eordinal_2Eordinal \\ & A_27a).((\neg(p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V6y) \ V5x))) \Rightarrow \\ & (\neg(p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ (ap \ V0f \ V6y)) \ (ap \ V0f \ V5x)))))) \wedge \\ & ((p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V1a1) \ V2a2)) \wedge ((\neg(p \ (ap \\ & (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V3c) \ (ap \ V0f \ V1a1)))) \wedge (p \ (ap \ (ap \\ & (c_2Eordinal_2Eordlt \ A_27a) \ V3c) \ (ap \ V0f \ V2a2)))))) \Rightarrow (\exists V7b \in \\ & (ty_2Eordinal_2Eordinal \ A_27a).((\neg(p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \\ & A_27a) \ V7b) \ V1a1))) \wedge ((p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V7b) \\ & V2a2)) \wedge ((\neg(p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V3c) \ (ap \ V0f \ V7b)))) \wedge \\ & (p \ (ap \ (ap \ (c_2Eordinal_2Eordlt \ A_27a) \ V3c) \ (ap \ V0f \ (ap \ (c_2Eordinal_2EordSUC \\ & A_27a) \ V7b)))))))))) \end{aligned}$$