

# thm\_2Eordinal\_2Eordle\_EXISTS\_ADD (TMN- Brv1qWqSwVmdf5J3XfQeBD43ZUuvVjLi)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 4** We define `c_2Ebool_2E_2IN` to be  $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \quad (1)$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c\_2Epair\_2EABS\_prod } A. 27a \ A. 27b \in ((\text{ty\_2Epair\_2Eprod } A. 27a \ A. 27b))^{(2^{A-27b})^{A-27a}} \quad (2)$$

**Definition 8** We define `c_2Epair_2E_2C` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap } (\text{c\_2Epair\_2EABS\_prod } A. 27a \ A. 27b))$

Let `c_2Epred_set_2EGSPEC` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c\_2Epred\_set\_2EGSPEC } A. 27a \ A. 27b \in ((2^{A-27a})^{(\text{ty\_2Epair\_2Eprod } A. 27a \ 2)^{A-27b}}) \quad (3)$$

**Definition 9** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($   
Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (4)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (5)$$

Let  $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \quad (6)$$

Let  $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \quad (7)$$

Let  $c\_2Eordinal\_2Eordinal\_REP\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27a \in ((\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_REP\_CLASS\ A\_27a) \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})) \quad (8)$$

**Definition 10** We define  $c\_2Eordinal\_2Eordinal\_REP$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let  $c\_2Ewellorder\_2Ewellorder\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$A\_27a \in ((\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_REP\ A\_27a) \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)})) \quad (9)$$

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 12** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E))$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (10)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (11)$$

**Definition 13** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})^{A\_27b})$

**Definition 14** We define  $c\_Eset\_relation\_Estrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

**Definition 15** We define  $c\_Ewellorder\_Eiseg$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 16** We define  $c\_Eset\_relation\_Erestrict$  to be  $\lambda A\_27a : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

Let  $c\_2Ewellorder\_2Ewellorder\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_ABS \\ & A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \end{aligned} \quad (12)$$

**Definition 17** We define  $c\_2Ewellorder\_2Ewobound$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1w \in (ty\_2Ewellorder\ A\_27a)$

**Definition 18** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A\_27a\ P))))$

**Definition 19** We define  $c\_Eset\_relation\_Erange$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 20** We define  $c\_Eset\_relation\_Edomain$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$

**Definition 21** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ t1\ t2)))$

**Definition 22** We define  $c\_2Epred\_set\_2EUNION$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{A\_27a}). \lambda V1t \in (2^{A\_27a}). (ap\ (c\_2Epred\_set\_2EUNION\ s\ t))$

**Definition 23** We define  $c\_2Ewellorder\_2EelsOf$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 24** We define  $c\_2Ewellorder\_2Eorderiso$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 25** We define  $c\_2Ewellorder\_2Eorderlt$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a)$

**Definition 26** We define  $c\_2Eordinal\_2Eordlt$  to be  $\lambda A\_27a : \iota. \lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

**Definition 27** We define  $c\_2Eordinal\_2Epreds$  to be  $\lambda A\_27a : \iota. \lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

**Definition 28** We define  $c\_2Epred\_set\_2EBIGUNION$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(2^{A\_27a})}). (ap\ (c\_2Epred\_set\_2EBIGUNION\ P))$

**Definition 29** We define  $c\_2Eordinal\_2Eoleast$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})$

**Definition 30** We define  $c\_2Eordinal\_2Esup$  to be  $\lambda A\_27a : \iota. \lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (13)$$

**Definition 31** We define  $c\_2Eone\_2Eone$  to be  $(ap\ (c\_2Emin\_2E\_40\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2Eone))$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_sum \\ & A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \end{aligned} \quad (14)$$

**Definition 32** We define  $c\_Esum\_2EINR$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27b. (ap (c\_Esum\_2EABS$   
Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty\_2Eoption\_2Eoption A0) \quad (15)$$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (16)$$

**Definition 33** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota. (ap (c\_2Eoption\_2Eoption\_ABS A\_27a))$

**Definition 34** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. c\_2Ebool\_2EF)$ .

**Definition 35** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2EF$

**Definition 36** We define  $c\_2Eset\_relation\_2Emaximal\_elements$  to be  $\lambda A\_27a : \iota. \lambda V0xs \in (2^{A\_27a}). \lambda V1r \in$

**Definition 37** We define  $c\_Esum\_2EINL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0e \in A\_27a. (ap (c\_Esum\_2EABS$

**Definition 38** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. (ap (c\_2Eoption\_2Eoption\_ABS$

**Definition 39** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. (c\_2Ebool\_2EF$

**Definition 40** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota. \lambda V0P \in (2^{A\_27a}). (ap (ap (ap (c\_2Ebool\_2ECOND$

**Definition 41** We define  $c\_2Eordinal\_2Eomax$  to be  $\lambda A\_27a : \iota. \lambda V0s \in (2^{(ty\_2Eordinal\_2Eordinal A\_27a)}). (ap (c\_2Eordinal\_2Eord$

**Definition 42** We define  $c\_2Eordinal\_2EordSUC$  to be  $\lambda A\_27a : \iota. \lambda V0a \in (ty\_2Eordinal\_2Eordinal A\_27a)$

Let  $c\_2Eordinal\_2EordADD : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eordinal\_2EordADD A\_27a \in ((ty\_2Eordinal\_2Eordinal A\_27a)^{(ty\_2Eordinal\_2Eordinal A\_27a)})^{(ty\_2Eordinal\_2Eordinal A\_27a)} \quad (17)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (18)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (19)$$

**Definition 43** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_2Eordinal\_2EfromNat A\_27a \in (ty\_2Eordinal\_2Eordinal A\_27a)^{ty\_2Enum\_2Enum} \quad (20)$$

Assume the following.

$$True \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (23)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (26)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p (ap V0P V3x))) \wedge (\forall V4x \in A\_27a.(p (ap V1Q V4x))))))) \quad (28)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A\_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (29)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p V0P) \wedge (\forall V2x \in A\_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (30)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A.27a. (p\ (ap\ V1Q\ V3x)))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee (p\ V1B)) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (32)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A.27a). (\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A.27a). ((\neg (p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A.27a)\ V0b)\ V1a))) \Leftrightarrow ((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A.27a)\ V1a)\ V0b)) \vee (V1a = V0b)))))) \quad (33)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A.27a). (((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0b)\ (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ c\_2Enum\_2E0)) = V0b) \wedge ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A.27a). ((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0b)\ (ap\ (c\_2Eordinal\_2EordSUC\ A.27a)\ V1a)) = (ap\ (c\_2Eordinal\_2EordSUC\ A.27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0b)\ V1a)))) \wedge (\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A.27a). (((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A.27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ c\_2Enum\_2E0))\ V2a)) \wedge ((ap\ (c\_2Eordinal\_2Eomax\ A.27a)\ (ap\ (c\_2Eordinal\_2Epreds\ A.27a)\ V2a)) = (c\_2Eoption\_2ENONE\ (ty\_2Eordinal\_2Eordinal\ A.27a)))))) \Rightarrow ((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0b)\ V2a) = (ap\ (c\_2Eordinal\_2Esup\ A.27a)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ (ty\_2Eordinal\_2Eordinal\ A.27a)\ (ty\_2Eordinal\_2Eordinal\ A.27a))\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0b))\ (ap\ (c\_2Eordinal\_2Epreds\ A.27a)\ V2a)))))) \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A.27a). (\forall V1b \in (ty\_2Eordinal\_2Eordinal\ A.27a). ((p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A.27a)\ V0a)\ V1b)) \Leftrightarrow (\exists V2c \in (ty\_2Eordinal\_2Eordinal\ A.27a). ((\neg (V2c = (ap\ (c\_2Eordinal\_2EfromNat\ A.27a)\ c\_2Enum\_2E0))) \wedge (V1b = (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A.27a)\ V0a)\ V2c)))))) \quad (35)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg (p\ V0A)) \Rightarrow False))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (49)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (50)$$

**Theorem 1**

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal A\_27a).(\forall V1b \in (ty\_2Eordinal\_2Eordinal A\_27a).((\neg(p (ap (ap (c\_2Eordinal\_2Eordlt A\_27a) V1b) V0a))) \Leftrightarrow (\exists V2c \in (ty\_2Eordinal\_2Eordinal A\_27a).(V1b = (ap (ap (c\_2Eordinal\_2EordADD A\_27a) V0a) V2c)))))))$$