

# thm\_2Eordinal\_2Eordlet\_\_TRANS (TMPx- ihY4aPfi3MXG4e9HwLMXMmRUepoRwbj)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let  $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{4}$$

Let  $c\_2Ewellorder\_2Ewellorder\_REP : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) (ty\_2Ewellorder\_2Ewellorder\ A\_27a)) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_2Emin\_2E3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E21\ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 5** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

**Definition 7** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS\_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

**Definition 8** We define `c_2Epair_2E_2C` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2E_2C))$

**Definition 9** We define `c_2Ebool_2EIN` to be  $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap V1f V0x)))$

Let `c_2Epair_2ESND` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let `c_2Epair_2EFST` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

**Definition 10** We define `c_2Epair_2EUNCURRY` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let `c_2Epred_set_2EGSPEC` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred\_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

**Definition 11** We define `c_2Eset_relation_2Estrict` to be  $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

**Definition 12** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (the (\lambda x. x \in A) p)$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40))))$

**Definition 14** We define `c_2Eset_relation_2Erange` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

**Definition 15** We define `c_2Eset_relation_2Edomain` to be  $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

**Definition 16** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))))$

**Definition 17** We define `c_2Epred_set_2EUNION` to be  $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2Epred\_set_2EUNION))$

**Definition 18** We define `c_2Ewellorder_2EelsOf` to be  $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

**Definition 19** We define  $c\_2Ewellorder\_2Eorderiso$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Let  $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \quad (10)$$

Let  $c\_2Eordinal\_2Eordinal\_ABS\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_ABS\_CLASS\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})}) \quad (11)$$

**Definition 20** We define  $c\_2Eordinal\_2Eordinal\_ABS$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ewellorder\_2Ewellorder$

Let  $c\_2Eordinal\_2Eordinal\_REP\_CLASS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_REP\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \quad (12)$$

**Definition 21** We define  $c\_2Eordinal\_2Eordinal\_REP$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A$

**Definition 22** We define  $c\_2Ewellorder\_2Eiseg$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

**Definition 23** We define  $c\_2Eset\_relation\_2Errestrict$  to be  $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27$

Let  $c\_2Ewellorder\_2Ewellorder\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (13)$$

**Definition 24** We define  $c\_2Ewellorder\_2Ewobound$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

**Definition 25** We define  $c\_2Ewellorder\_2Eorderlt$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

**Definition 26** We define  $c\_2Eordinal\_2Eordlt$  to be  $\lambda A\_27a : \iota.\lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

**Definition 27** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

**Definition 28** We define  $c\_2Ecombin\_2ES$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27$

**Definition 29** We define  $c\_2Ecombin\_2EI$  to be  $\lambda A\_27a : \iota.(ap\ (ap\ (c\_2Ecombin\_2ES\ A\_27a\ (A\_27a^{A\_27a})\ A$

**Definition 30** We define  $c\_2Equotient\_2E\_2D\_2D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0j$

**Definition 31** We define  $c\_2Equotient\_2E\_3D\_3D\_3D\_3E$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27$

**Definition 32** We define  $c\_2\text{Equotient\_2EQUOTIENT}$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

**Definition 33** We define  $c\_2\text{Ecombin\_2EW}$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

**Definition 34** We define  $c\_2\text{Equotient\_2Erespects}$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(c\_2\text{Ecombin\_2EW } A\_27a \ A\_27b$

**Definition 35** We define  $c\_2\text{Ebool\_2ERES\_FORALL}$  to be  $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).$

**Definition 36** We define  $c\_2\text{Equotient\_2EEQUIV}$  to be  $\lambda A\_27a : \iota.\lambda V0E \in ((2^{A\_27a})^{A\_27a}).(ap \ (c\_2\text{Ebool\_2E}$

Assume the following.

$$\text{True} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow \text{True}) \Leftrightarrow \\ & \text{True}) \wedge (((\text{False} \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow \text{True}) \wedge (( \\ & (p \ V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg \text{True}) \Leftrightarrow \text{False}) \wedge \\ & ((\neg \text{False}) \Leftrightarrow \text{True})))) \end{aligned} \quad (16)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \end{aligned} \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg( \\ & p \ V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p \ V0A) \vee ( \\ & (p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (22)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2}Ecombin_{.2EI} A_{.27a}) V0x) = V0x)) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} (ty_{.2}Ewellorder_{.2Ewellorder} (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (ty_{.2}Eordinal_{.2Eordinal} A_{.27a})) (c_{.2}Ewellorder_{.2Eorderiso} (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (c_{.2}Eordinal_{.2Eordinal\_ABS} A_{.27a})) (c_{.2}Eordinal_{.2Eordinal\_REP} A_{.27a}))) \quad (25)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0b \in (ty_{.2}Eordinal_{.2Eordinal} A_{.27a}).(\forall V1a \in (ty_{.2}Eordinal_{.2Eordinal} A_{.27a}).(\neg (p (ap (ap (c_{.2}Eordinal_{.2Eordlt} A_{.27a}) V0b) V1a))) \Leftrightarrow ((p (ap (ap (c_{.2}Eordinal_{.2Eordlt} A_{.27a}) V1a) V0b)) \vee (V1a = V0b)))))) \quad (26)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27a} A_{.27a}) (c_{.2}Emin_{.2E_{.3D}} A_{.27a})) (c_{.2}Ecombin_{.2EI} A_{.27a})) (c_{.2}Ecombin_{.2EI} A_{.27a}))) \quad (27)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow \forall A_{.27d}.nonempty A_{.27d} \Rightarrow (\forall V0R1 \in (2^{A_{.27a}^{A_{.27a}}}).(\forall V1abs1 \in (A_{.27c}^{A_{.27a}}).(\forall V2rep1 \in (A_{.27a}^{A_{.27c}}).((p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27a} A_{.27c}) V0R1) V1abs1) V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}^{A_{.27b}}}).(\forall V4abs2 \in (A_{.27d}^{A_{.27b}}).(\forall V5rep2 \in (A_{.27b}^{A_{.27d}}).((p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27b} A_{.27d}) V3R2) V4abs2) V5rep2)) \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} (A_{.27b}^{A_{.27a}}) (A_{.27d}^{A_{.27c}})) (ap (ap (c_{.2}Equotient_{.2E_{.3D}} A_{.27a} A_{.27b}) V0R1) V3R2)) (ap (ap (c_{.2}Equotient_{.2E_{.2D}} A_{.27c} A_{.27b} A_{.27a} A_{.27d}) V2rep1) V4abs2)) (ap (ap (c_{.2}Equotient_{.2E_{.2D}} A_{.27a} A_{.27d} A_{.27c} A_{.27b}) V1abs1) V5rep2)))))))))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). \\
& \quad ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_2D\_2D\_3E \\
& \quad A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in \\
& \quad A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ ( \\
& \quad ap\ (c\_2Ebool\_2E\_21\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c\_2Equotient\_2E\_2D\_2D\_3E\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (c\_2Ecombin\_2EI \\
& \quad 2))\ V3f)))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). \\
& \quad (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in \\
& \quad (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad 2)\ V0R)\ (c\_2Emin\_2E\_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL \\
& \quad A\_27a)\ (ap\ (c\_2Equotient\_2Erespects\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ ( \\
& \quad ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0R))\ V4g)))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\
& \quad nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ( \\
& \quad (2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in \\
& \quad (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT\ A\_27a\ A\_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in \\
& \quad (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (c\_2Equotient\_2EQUOTIENT \\
& \quad A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). \\
& \quad (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in \\
& \quad A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c\_2Equotient\_2E\_3D\_3D\_3D\_3E\ A\_27a \\
& \quad A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ ( \\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). \\
& \quad (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (c\_2Equotient\_2EEQUIV\ A\_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c\_2Ebool\_2ERES\_FORALL\ A\_27a)\ (ap\ (c\_2Equotient\_2Erespects \\
& \quad A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c\_2Ebool\_2E\_21\ A\_27a)\ V1P))))))
\end{aligned} \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{35}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee \neg(p \ V2r))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge ((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow ( \\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ( \\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee \neg(p \ V0p)))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q)) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q)) \vee \neg(p \ V0p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{45}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q)))) \tag{46}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V0p)))) \tag{47}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \vee (p \ V1q))) \Rightarrow \neg(p \ V1q)))) \tag{48}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p \ V0p))) \Rightarrow (p \ V0p))) \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0w \in (ty\_2Ewellorder\_2Ewellorder \\
& A\_27a).(p \ (ap \ (ap \ (c\_2Ewellorder\_2Eorderiso \ A\_27a \ A\_27a) \ V0w) \\
& V0w)))
\end{aligned} \tag{50}$$



Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \quad \forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in \\ & (ty\_2Ewellorder\_2Ewellorder\ A\_27b).((p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderiso \\ & \quad A\_27a\ A\_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderiso \\ & \quad A\_27b\ A\_27a)\ V1w2)\ V0w1)))))) \end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). \\ & \quad (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in \\ & (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderiso \\ & \quad A\_27a\ A\_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderiso \\ & \quad A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderiso \\ & \quad A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a). \\ & \quad (\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in \\ & (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderlt \\ & \quad A\_27a\ A\_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderlt \\ & \quad A\_27b\ A\_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c\_2Ewellorder\_2Eorderlt \\ & \quad A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0x0 \in ( \\ & ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1y0 \in (ty\_2Ewellorder\_2Ewellorder \\ & \quad A\_27b).(\forall V2a0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c). \\ & \quad (\forall V3b0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27d).(((p\ (ap\ (ap \\ & \quad (c\_2Ewellorder\_2Eorderiso\ A\_27a\ A\_27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap \\ & \quad (c\_2Ewellorder\_2Eorderiso\ A\_27c\ A\_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap \\ & \quad (ap\ (c\_2Ewellorder\_2Eorderlt\ A\_27a\ A\_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap \\ & \quad (ap\ (c\_2Ewellorder\_2Eorderlt\ A\_27b\ A\_27d)\ V1y0)\ V3b0)))))) \end{aligned} \tag{54}$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal \\ & \quad A\_27a).(\forall V1y \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2z \in \\ & (ty\_2Eordinal\_2Eordinal\ A\_27a).(((\neg(p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt \\ & \quad A\_27a)\ V1y)\ V0x))) \wedge (p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A\_27a)\ V1y) \\ & \quad V2z)))) \Rightarrow (p\ (ap\ (ap\ (c\_2Eordinal\_2Eordlt\ A\_27a)\ V0x)\ V2z)))))) \end{aligned}$$