thm_2Eordinal_2Eordlt__EXISTS__ADD (TMHoBUP-pSD68uyQMMT51p76YSLVi6uRHm6j)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{3}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{4}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{6}$$

Definition 9 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Definition 10 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{8}$$

Definition 11 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{9}$$

Definition 12 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

Definition 13 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 14 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 15 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 16 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c.$

Definition 18 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A$

Definition 19 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2$

Let $ty\_2Eordinal\_2Eordinal : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \quad (10)$$

Let $c\_2Eordinal\_2Eordinal\_\_ABS\_\_CLASS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eordinal\_2Eordinal\_\_ABS\_\_CLASS\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})}) \quad (11)$$

Definition 20 *We define* c_2Eordinal_2Eordinal__ABS *to be* $\lambda A\_27a : \iota.\lambda V0r \in (ty\_2Ewellorder\_2Ewellorde$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \quad (12)$$

Definition 21 *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A$

Definition 22 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Definition 23 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 24 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (13)$$

Definition 25 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

Definition 26 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Definition 27 *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a : \iota.\lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

Definition 28 *We define* c_2Eordinal_2Epreds *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).($

Definition 29 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 30 *We define* c_2Eordinal_2Eoleast *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap$

Definition 31 *We define* c_2Eordinal_2Esup *to be* $\lambda A\_27a : \iota.\lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).($

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (14)$$

Definition 32 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \quad (15)$$

Definition 33 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \quad (16)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (17)$$

Definition 34 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 35 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 36 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c$

Definition 37 *We define* c_2Eset__relation_2Emaximal__elements *to be* $\lambda A\_27a : \iota.\lambda V0xs \in (2^{A\_27a}).\lambda V1r \in$

Definition 38 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Definition 39 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

Definition 40 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 41 *We define* c_2Eoption_2Esome *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EC}$

Definition 42 *We define* c_2Eordinal_2Eomax *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap\ ($

Definition 43 *We define* c_2Eordinal_2EordSUC *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum}) \quad (18)$$

Let $c\_2Eordinal\_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EordADD\ A\_27a \in (((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}) \quad (19)$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{20}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{21}$$

Definition 44 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{22}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{23}$$

Definition 45 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 46 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 47 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x$

Definition 48 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 49 *We define* c_2Ecombin_2EI *to be* $\lambda A_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Definition 50 *We define* c_2Equotient_2E_2D_2D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda A\_27d : \iota.\lambda V0f$

Definition 51 *We define* c_2Equotient_2E_3D_3D_3D_3E *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R1 \in ((2^{A\_27a})^{A\_27}$

Definition 52 *We define* c_2Equotient_2EQUOTIENT *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda$

Definition 53 *We define* c_2Ecombin_2EW *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in ((A\_27b^{A\_27a})^{A\_27a}).(\lambda V1x$

Definition 54 *We define* c_2Equotient_2Erespects *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.($c_2Ecombin_2EW $A\_27a\ A\_2$

Definition 55 *We define* c_2Ebool_2ERES__FORALL *to be* $\lambda A\_27a : \iota.(\lambda V0p \in (2^{A\_27a}).(\lambda V1m \in (2^{A\_27a}).($

Definition 56 *We define* c_2Equotient_2EEQUIV *to be* $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap\ ($c_2Ebool_2E

Assume the following.

$$True \tag{24}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{29}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \Rightarrow (p\ V1Q))))) \tag{36}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee (p\ V0A))))) \tag{38}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg (p\ V0A)) \vee (p\ V1B))))) \tag{39}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{40}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \tag{41}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (ap\ V0f\ V1v))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))\ (\mathsf{c\_2Eordinal\_2Eordinal\_\_ABS}\ A\_27a))\ (\mathsf{c\_2Eordinal\_2Eordinal\_\_REP}\ A\_27a))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0w)\ V0w)))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0b)\ V1a))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V0b)) \lor (V1a = V0b))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))))) \tag{49}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1b))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ V1b)) \lor (V0a = V1b))))) \tag{50}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2b)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0f)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1a))))) \Leftrightarrow (\exists V3d \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3d)\ V1a)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2b)\ (ap\ V0f\ V3d))))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}). (((p\ (ap\ V0P\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})))\wedge ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ V0P\ V1a))\Rightarrow (p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a)))))\wedge(\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)))\wedge((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a))\wedge (\forall V3b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3b)\ V2a))\Rightarrow(p\ (ap\ V0P\ V3b))))))\Rightarrow(p\ (ap\ V0P\ V2a))))))\Rightarrow(\forall V4a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ V0P\ V4a)))))\tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ V0n))\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ V1m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m)))))\tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0})) = V0b)\wedge((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a)) = (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ V1a))))\wedge(\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a). (((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a))\wedge((ap\ (\mathsf{c\_2Eordinal\_2Eomax}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))))\Rightarrow((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b)\ V2a) = (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0b))\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)))))))))\tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V1a)\ V0b)))\Leftrightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V0b)))))\tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2c \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V1a)\ V0b) = (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V1a)\ V2c)) \Leftrightarrow (V0b = V2c))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V1a)\ V0x))\ V0x))))) \tag{57}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27a)\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ (A\_27b^{A\_27a})\ (A\_27d^{A\_27c}))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ A\_27d\ A\_27c\ A\_27b)\ V1abs1)\ V5rep2)))))))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}).(\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x \in A\_27b.(\forall V4y \in A\_27b.((V3x = V4y) \Leftrightarrow (p\ (ap\ (ap\ V0R\ (ap\ V2rep\ V3x))\ (ap\ V2rep\ V4y)))))))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in A\_27a.(\forall V5y1 \in A\_27a.(\forall V6y2 \in A\_27a.(((p\ (ap\ (ap\ V0R\ V3x1)\ V4x2)) \wedge (p\ (ap\ (ap\ V0R\ V5y1)\ V6y2))) \Rightarrow ((p\ (ap\ (ap\ V0R\ V3x1)\ V5y1)) \Leftrightarrow (p\ (ap\ (ap\ V0R\ V4x2)\ V6y2)))))))))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27d^{A\_27c}). ((\lambda V7x \in A\_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27c\ A\_27b\ A\_27a\ A\_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A\_27a.(ap\ V5rep2\ (ap\ V6f\ (ap\ V1abs1\ V8x)))))))))))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0REL \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A\_27a.(\forall V4x2 \in A\_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep\ (ap\ V1abs\ V4x2)))))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27b}).((p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_2D\_2D\_3E}\ A\_27a\ 2\ A\_27b\ 2)\ V1abs)\ (\mathsf{c\_2Ecombin\_2EI}\ 2))\ V3f))))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs \in (A\_27b^{A\_27a}). (\forall V2rep \in (A\_27a^{A\_27b}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A\_27a}).(\forall V4g \in (2^{A\_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ 2)\ V0R)\ (\mathsf{c\_2Emin\_2E\_3D}\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0R))\ V4g))))))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0R1 \in ((2^{A\_27a})^{A\_27a}).(\forall V1abs1 \in (A\_27c^{A\_27a}).(\forall V2rep1 \in (A\_27a^{A\_27c}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27a\ A\_27c)\ V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A\_27b})^{A\_27b}).(\forall V4abs2 \in (A\_27d^{A\_27b}).(\forall V5rep2 \in (A\_27b^{A\_27d}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2EQUOTIENT}\ A\_27b\ A\_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A\_27b^{A\_27a}). (\forall V7g \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.(\forall V9y \in A\_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (\mathsf{c\_2Equotient\_2E\_3D\_3D\_3D\_3E}\ A\_27a\ A\_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \land (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))))))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0E \in ((2^{A\_27a})^{A\_27a}). (\forall V1P \in (2^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Equotient\_2EEQUIV}\ A\_27a)\ V0E)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ERES\_\_FORALL}\ A\_27a)\ (ap\ (\mathsf{c\_2Equotient\_2Erespects}\ A\_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_21}\ A\_27a)\ V1P)))))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{71}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \lor (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{72}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{73}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee((\neg(p\ V1q))\vee(\neg(p\ V0p))))))))))) \tag{74}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee(\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p))))))))) \tag{75}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V0p))))) \tag{81}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\vee(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p)))\Rightarrow(p\ V0p))) \tag{83}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27a)\ V0w)\ V0w))) \tag{84}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0w1)\ V1w2))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27b\ A\_27a)\ V1w2)\ V0w1))))) \tag{85}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0w1)\ V1w2))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27b\ A\_27c)\ V1w2)\ V2w3)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \tag{86}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27a)\ V0w)\ V0w))\Leftrightarrow False)) \tag{87}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.nonempty\ A\_27c\Rightarrow(\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2w3 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27b)\ V0w1)\ V1w2))\wedge(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27b\ A\_27c)\ V1w2)\ V2w3)))\Rightarrow(p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27c)\ V0w1)\ V2w3)))))) \tag{88}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0w1 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1w2 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \lor ((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0w1)\ V1w2)) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27b\ A\_27a)\ V1w2)\ V0w1)))))) \tag{89}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0x0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).(\forall V1y0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27b).(\forall V2a0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27c).(\forall V3b0 \in (ty\_2Ewellorder\_2Ewellorder\ A\_27d).(((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27a\ A\_27b)\ V0x0)\ V1y0)) \land (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderiso}\ A\_27c\ A\_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27a\ A\_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Eorderlt}\ A\_27b\ A\_27d)\ V1y0)\ V3b0)))))))) \tag{90}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ V1b)) \Leftrightarrow (\exists V2c \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((\neg(V2c = (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \land (V1b = (ap\ (ap\ (c\_2Eordinal\_2EordADD\ A\_27a)\ V0a)\ V2c)))))))$$