

thm_2Eordinal_2Epolyform__0 (TMSMxfVjYdBuscT8DuNvejapjDGc7XnJgwG)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Emarker_2E_Cong$ to be $\lambda V0x \in 2.V0x$.

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{1}$$

Definition 6 We define c_2Eone_2Eone to be $(ap (c_2Emin_2E_40 ty_2Eone_2Eone) (\lambda V0x \in ty_2Eone_2Eone$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_27E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $c_2Esum_2EABS_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Esum_2EABS_sum\ A_27a\ A_27b \in ((ty_2Esum_2Esum\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})^2}) \tag{3}$$

Definition 11 We define c_Esum_2EINR to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0e \in A_27b. (ap (c_Esum_2EABS$
Let $ty_2Eoption_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eoption_2Eoption A0) \quad (4)$$

Let $c_2Eoption_2Eoption_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eoption_2Eoption_ABS A_27a \in ((ty_2Eoption_2Eoption A_27a)^{(ty_2Esum_2Esum A_27a ty_2Eone_2Eone)}) \quad (5)$$

Definition 12 We define $c_2Eoption_2ENONE$ to be $\lambda A_27a : \iota. (ap (c_2Eoption_2Eoption_ABS A_27a) (c_2Eone_2Eone$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty ty_2Eenum_2Eenum \quad (6)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Ewellorder_2Ewellorder A0) \quad (7)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow nonempty (ty_2Eordinal_2Eordinal A0) \quad (8)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Eenum_2Eenum A_27a))})^{(ty_2Eordinal_2Eordinal A_27a)}) \quad (9)$$

Definition 13 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. nonempty A0 \Rightarrow \forall A1. nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (10)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)}) \quad (11)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (12)$$

Definition 14 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2Epair_2EABS_prod$

Definition 15 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}). (ap\ V1f\ V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (13)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (14)$$

Definition 16 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c^{A_27a})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (15)$$

Definition 17 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 18 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 19 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \end{aligned} \quad (16)$$

Definition 20 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder\ A_27a)$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a\ V0P))))$

Definition 22 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 23 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 24 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_3F\ A_27a\ V2t)\ V1t2))\ V0t1)))$

Definition 25 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Emin_2E_40\ A_27a\ V1t)\ V0s)$

Definition 26 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 27 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 28 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 29 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a).$

Definition 30 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal A_27a).$

Definition 31 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 32 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap (c_2Ebool_2EF).$

Definition 33 We define $c_2Eset_relation_2Emaximal_elements$ to be $\lambda A_27a : \iota.\lambda V0xs \in (2^{A_27a}).\lambda V1r \in$

Definition 34 We define c_2Esum_2EINL to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0e \in A_27a.(ap (c_2Esum_2EABS$

Definition 35 We define $c_2Eoption_2ESOME$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.(ap (c_2Eoption_2EOption$

Definition 36 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 37 We define $c_2Eoption_2ESome$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).(ap (ap (ap (c_2Ebool_2ECOND$

Definition 38 We define $c_2Eordinal_2Eomax$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Let $c_2Eordinal_2EordEXP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2EordEXP A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}(ty_2Eordinal_2Eordinal A_27a)) \quad (17)$$

Let $c_2Eordinal_2EordMULT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2EordMULT A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}(ty_2Eordinal_2Eordinal A_27a)) \quad (18)$$

Let $c_2Eordinal_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2EordADD A_27a \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}(ty_2Eordinal_2Eordinal A_27a)) \quad (19)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (20)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (21)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (22)$$

Let $c_2Elist_2ELIST_TO_SET : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELIST_TO_SET\ A_27a \in ((2^{A_27a})^{(ty_2Elist_2Elist\ A_27a)}) \quad (23)$$

Let $c_2Elist_2EEL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEL\ A_27a \in ((A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{ty_2Enum_2Enum} \quad (24)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Elist_2Elist\ A_27a)} \quad (25)$$

Let $c_2Eordinal_2Eval_poly : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eval_poly\ A_27a \in (((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal\ A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a))}))^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (26)$$

Let $c_2Eordinal_2Epolyform : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Epolyform\ A_27a \in (((ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal\ A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a)))^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (27)$$

Let $c_2Eordinal_2Eis_polyform : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Eordinal_2Eis_polyform\ A_27a\ A_27b \in ((2^{(ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal\ A_27a)\ (ty_2Eordinal_2Eordinal\ A_27b)))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (28)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (29)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (30)$$

Definition 39 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 40 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (31)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (32)$$

Definition 41 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ (c_2Enum_2EREP_num\ V0m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 42 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 43 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Eordinal_2EfromNat A.27a \in (ty_2Eordinal_2Eordinal A.27a)^{ty_2Enum_2Enum} \quad (34)$$

Definition 44 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \quad (39)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \quad (41)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t)))))) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in (2^{A.27a}). ((\forall V2x \in A.27a. ((p (ap V0P V2x)) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((\forall V3x \in A.27a. (p (ap V0P V3x))) \wedge (\forall V4x \in A.27a. (p (ap V1Q V4x))))))) \quad (45)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in 2. (((\forall V2x \in A.27a. (p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A.27a. ((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (46)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((p V0P) \wedge (\forall V2x \in A.27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A.27a. ((p V0P) \wedge (p (ap V1Q V3x)))))) \quad (47)$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A.27a}). (((\forall V2x \in A.27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A.27a. (p (ap V1Q V3x)))))) \quad (48)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (49)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (51)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (52)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0l \in (ty_{.2Elist}_{.2Elist} A_{.27a}).((V0l = (c_{.2Elist}_{.2ENIL} A_{.27a})) \vee (\exists V1h \in A_{.27a}.(\exists V2t \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(V0l = (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V1h) V2t)))))) \Rightarrow (53)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a1 \in (ty_{.2Elist}_{.2Elist} A_{.27a}).(\forall V1a0 \in A_{.27a}.(\neg((c_{.2Elist}_{.2ENIL} A_{.27a}) = (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V1a0) V0a1)))) \Rightarrow (54)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0x \in A_{.27a}.((p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V0x) (ap (c_{.2Elist}_{.2ELIST_TO_SET} A_{.27a}) (c_{.2Elist}_{.2ENIL} A_{.27a})))) \Leftrightarrow False)) \wedge (\forall V1x \in A_{.27a}.(\forall V2h \in A_{.27a}.(\forall V3t \in (ty_{.2Elist}_{.2Elist} A_{.27a}).((p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V1x) (ap (c_{.2Elist}_{.2ELIST_TO_SET} A_{.27a}) (ap (ap (c_{.2Elist}_{.2ECONS} A_{.27a}) V2h) V3t)))) \Leftrightarrow ((V1x = V2h) \vee (p (ap (ap (c_{.2Ebool}_{.2EIN} A_{.27a}) V1x) (ap (c_{.2Elist}_{.2ELIST_TO_SET} A_{.27a}) V3t)))))))))) \Rightarrow (55)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in ty_{.2Enum}_{.2Enum}.(\forall V1y \in ty_{.2Enum}_{.2Enum}.(((ap (c_{.2Eordinal}_{.2EfromNat} A_{.27a}) V0x) = (ap (c_{.2Eordinal}_{.2EfromNat} A_{.27a}) V1y)) \Leftrightarrow (V0x = V1y)))) \Rightarrow (56)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0n \in ty_{.2Enum}_{.2Enum}.(\forall V1m \in ty_{.2Enum}_{.2Enum}.((p (ap (ap (c_{.2Eordinal}_{.2Eordlt} A_{.27a}) (ap (c_{.2Eordinal}_{.2EfromNat} A_{.27a}) V0n)) (ap (c_{.2Eordinal}_{.2EfromNat} A_{.27a}) V1m))) \Leftrightarrow (p (ap (ap (c_{.2Eprim_rec}_{.2E.3C} V0n) V1m)))))) \Rightarrow (57)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\
& A_27a).((\neg(V0x = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))) \Leftrightarrow \\
& (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0))\ V0x))) \wedge ((\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& A_27a)\ V0x)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \Leftrightarrow (p\ (\\
& ap\ (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0))\ V0x))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0y \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1x \in (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (\\
& ap\ (c_2Eordinal_2EordADD\ A_27a)\ V1x)\ V0y) = (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0)) \Leftrightarrow ((V1x = (ap\ (c_2Eordinal_2EfromNat\ A_27a) \\
& c_2Enum_2E0)) \wedge (V0y = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1y \in (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (\\
& ap\ (c_2Eordinal_2EordMULT\ A_27a)\ V0x)\ V1y) = (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0)) \Leftrightarrow ((V0x = (ap\ (c_2Eordinal_2EfromNat\ A_27a) \\
& c_2Enum_2E0)) \vee (V1y = (ap\ (c_2Eordinal_2EfromNat\ A_27a)\ c_2Enum_2E0))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0y \in (ty_2Eordinal_2Eordinal \\
& A_27a).(\forall V1x \in (ty_2Eordinal_2Eordinal\ A_27a).(((ap\ (\\
& ap\ (c_2Eordinal_2EordEXP\ A_27a)\ V1x)\ V0y) = (ap\ (c_2Eordinal_2EfromNat \\
& A_27a)\ c_2Enum_2E0)) \Leftrightarrow ((V1x = (ap\ (c_2Eordinal_2EfromNat\ A_27a) \\
& c_2Enum_2E0)) \wedge (\neg((ap\ (c_2Eordinal_2Eomax\ A_27a)\ (ap\ (c_2Eordinal_2Epreds \\
& A_27a)\ V0y)) = (c_2Eoption_2ENONE\ (ty_2Eordinal_2Eordinal\ A_27a)))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A.27a).((ap\ (ap\ (c_2Eordinal_2Eval_poly\ A.27a)\ V0a)\ (c_2Elist_2ENIL \\
& \quad (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal \\
& \quad \quad A.27a)))) = (ap\ (c_2Eordinal_2EfromNat\ A.27a)\ c_2Enum_2E0))) \wedge \\
& \quad (\forall V1t \in (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27a))).(\forall V2e \in (ty_2Eordinal_2Eordinal \\
& \quad \quad A.27a).(\forall V3c \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V4a \in \\
& \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a).((ap\ (ap\ (c_2Eordinal_2Eval_poly \\
& \quad \quad \quad A.27a)\ V4a)\ (ap\ (ap\ (c_2Elist_2ECONS\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad \quad \quad \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27a)))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27a)) \\
& \quad \quad \quad \quad V3c)\ V2e))\ V1t)) = (ap\ (ap\ (c_2Eordinal_2EordADD\ A.27a)\ (ap\ (ap\ (\\
& \quad \quad \quad \quad c_2Eordinal_2EordMULT\ A.27a)\ (ap\ (ap\ (c_2Eordinal_2EordEXP\ A.27a)\ \\
& \quad \quad \quad \quad V4a)\ V2e))\ V3c))\ (ap\ (ap\ (c_2Eordinal_2Eval_poly\ A.27a)\ V4a) \\
& \quad \quad \quad \quad V1t))))))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0a \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V1ces \in \\
& \quad \quad (ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad \quad \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b))).((p\ (ap\ (ap\ (c_2Eordinal_2Eis_polyform \\
& \quad \quad \quad A.27a\ A.27b)\ V0a)\ V1ces)) \Leftrightarrow ((\forall V2i \in ty_2Enum_2Enum.(\forall V3j \in \\
& \quad \quad \quad ty_2Enum_2Enum.(((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V2i)\ V3j)) \wedge (\\
& \quad \quad \quad p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V3j)\ (ap\ (c_2Elist_2ELENGTH\ (ty_2Epair_2Eprod \\
& \quad \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b))) \\
& \quad \quad \quad \quad V1ces)))) \Rightarrow (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27b)\ (ap\ (c_2Epair_2ESND \\
& \quad \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)) \\
& \quad \quad \quad \quad (ap\ (ap\ (c_2Elist_2EEL\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad \quad \quad \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)))\ V3j)\ V1ces)))\ (ap\ (c_2Epair_2ESND \\
& \quad \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)) \\
& \quad \quad \quad \quad (ap\ (ap\ (c_2Elist_2EEL\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad \quad \quad \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)))\ V2i)\ V1ces)))))) \wedge (\\
& \quad \forall V4c \in (ty_2Eordinal_2Eordinal\ A.27a).(\forall V5e \in (ty_2Eordinal_2Eordinal \\
& \quad \quad A.27b).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad \quad \quad \quad A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad \quad \quad \quad (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b)) \\
& \quad \quad \quad \quad V4c)\ V5e))\ (ap\ (c_2Elist_2ELIST_TO_SET\ (ty_2Epair_2Eprod\ (\\
& \quad \quad \quad \quad ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Eordinal_2Eordinal\ A.27b))) \\
& \quad \quad \quad \quad V1ces))) \Rightarrow ((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ (ap\ (c_2Eordinal_2EfromNat \\
& \quad \quad \quad \quad A.27a)\ c_2Enum_2E0))\ V4c)) \wedge (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a) \\
& \quad \quad \quad \quad V4c)\ V0a))))))))) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a).(\forall V1b \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap \\
& \quad (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ (ap\ (c_2Eordinal_2EfromNat\ A_27a) \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\
& \quad V0a)) \Rightarrow ((p\ (ap\ (ap\ (c_2Eordinal_2Eis_polyform\ A_27a\ A_27a)\ V0a) \\
& \quad (ap\ (ap\ (c_2Eordinal_2Epolyform\ A_27a)\ V0a)\ V1b))) \wedge (V1b = (ap\ (\\
& \quad ap\ (c_2Eordinal_2Eval_poly\ A_27a)\ V0a)\ (ap\ (ap\ (c_2Eordinal_2Epolyform \\
& \quad \quad A_27a)\ V0a)\ V1b))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a. \\
& \quad (\exists V2r \in A_27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\
& \quad \quad V1q)\ V2r))))))
\end{aligned} \tag{65}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
\quad V0n)\ c_2Enum_2E0)))) \tag{66}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{67}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{68}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
\quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \tag{69}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
\quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \tag{70}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{76}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{81}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) (ap (c_2Eordinal_2EfromNat \\
& A_27a) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) V0a) \Rightarrow ((ap (ap (c_2Eordinal_2Epolyform \\
& A_27a) V0a) (ap (c_2Eordinal_2EfromNat A_27a) c_2Enum_2E0)) = \\
& (c_2Elist_2ENIL (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal \\
& A_27a) (ty_2Eordinal_2Eordinal A_27a))))))
\end{aligned}$$