

thm_2Eordinal_2Epolyform__EQ__NIL (TMMCK-
 Tup6NngmkbsHiRb1APb5FrSaEWYc2KR)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \tag{2}$$

Let $c_2Eordinal_2EordEXP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordEXP\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \tag{3}$$

Let $c_2Eordinal_2EordMULT : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordMULT\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \tag{4}$$

Let $c_2Eordinal_2EordADD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EordADD\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Eordinal_2Eordinal\ A_27a)})^{(ty_2Eordinal_2Eordinal\ A_27a)} \tag{5}$$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2))$
Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (7)$$

Definition 8 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$
Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (8)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ECONS A_27a \in (((ty_2Elist_2Elist A_27a)^{(ty_2Elist_2Elist A_27a)})^{A_27a}) \quad (9)$$

Let $c_2Eordinal_2Eeval_poly : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Eordinal_2Eeval_poly A_27a \in (((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Elist_2Elist (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal A_27a) (ty_2Eordinal_2Eordinal A_27a))})^{A_27a})^{A_27a}) \quad (10)$$

Let $c_2Eordinal_2Eis_polyform : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Eordinal_2Eis_polyform A_27a A_27b \in ((2^{(ty_2Elist_2Elist (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal A_27a) (ty_2Eordinal_2Eordinal A_27b)))})^{A_27a})^{A_27b} \quad (11)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Elist_2ENIL A_27a \in (ty_2Elist_2Elist A_27a) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (14)$$

Definition 9 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Eordinal_2Epolyform : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Epolyform\ A_27a \in \\ & ((ty_2Elist_2Elist\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\ & A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a)))^{(ty_2Eordinal_2Eordinal\ A_27a)}\ (ty_2Eordinal_2Eordinal\ A_27a)) \end{aligned} \quad (15)$$

Definition 10 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (17)$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (18)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 13 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Eordinal_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2EfromNat\ A_27a \in (\\ & (ty_2Eordinal_2Eordinal\ A_27a)^{ty_2Enum_2Enum} \end{aligned} \quad (19)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum \\ & A0\ A1) \end{aligned} \quad (20)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (21)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS \\ & A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)} \end{aligned} \quad (22)$$

Definition 14 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 26 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c$

Definition 27 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A$

Definition 28 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 29 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 30 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal A_27a).$

Assume the following.

$$True \tag{28}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{30}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{31}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{32}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ & p V0t)))))) \end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ & V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \end{aligned} \tag{35}$$

Assume the following.

$$2.((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in ty_2Enum_2Enum.(\forall V1y \in ty_2Enum_2Enum.(((ap (c_2Eordinal_2EfromNat A_{.27a}) V0x) = (ap (c_2Eordinal_2EfromNat A_{.27a}) V1y)) \Leftrightarrow (V0x = V1y)))) \quad (37)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0a \in (ty_2Eordinal_2Eordinal A_{.27a}).((ap (ap (c_2Eordinal_2Eval_poly A_{.27a}) V0a) (c_2Elist_2ENIL (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal A_{.27a}) (ty_2Eordinal_2Eordinal A_{.27a})))) = (ap (c_2Eordinal_2EfromNat A_{.27a}) c_2Enum_2E0)))) \wedge (\forall V1t \in (ty_2Elist_2Elist (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal A_{.27a}) (ty_2Eordinal_2Eordinal A_{.27a}))).(\forall V2e \in (ty_2Eordinal_2Eordinal A_{.27a}).(\forall V3c \in (ty_2Eordinal_2Eordinal A_{.27a}).(\forall V4a \in (ty_2Eordinal_2Eordinal A_{.27a}).((ap (ap (c_2Eordinal_2Eval_poly A_{.27a}) V4a) (ap (ap (c_2Elist_2ECONS (ty_2Epair_2Eprod (ty_2Eordinal_2Eordinal A_{.27a}) (ty_2Eordinal_2Eordinal A_{.27a}))) (ap (ap (c_2Epair_2E_2C (ty_2Eordinal_2Eordinal A_{.27a}) (ty_2Eordinal_2Eordinal A_{.27a}))) V3c) V2e)) V1t)) = (ap (ap (c_2Eordinal_2EordADD A_{.27a}) (ap (ap (c_2Eordinal_2EordMULT A_{.27a}) (ap (ap (c_2Eordinal_2EordEXP A_{.27a}) V4a) V2e)) V3c)) (ap (ap (c_2Eordinal_2Eval_poly A_{.27a}) V4a) V1t)))))))) \quad (38)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal A_{.27a}).(\forall V1b \in (ty_2Eordinal_2Eordinal A_{.27a}).((p (ap (ap (c_2Eordinal_2Eordlt A_{.27a}) (ap (c_2Eordinal_2EfromNat A_{.27a}) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0a)) \Rightarrow ((p (ap (ap (c_2Eordinal_2Eis_polyform A_{.27a} A_{.27a}) V0a) (ap (ap (c_2Eordinal_2Epolyform A_{.27a}) V0a) V1b))) \wedge (V1b = (ap (ap (c_2Eordinal_2Eval_poly A_{.27a}) V0a) (ap (ap (c_2Eordinal_2Epolyform A_{.27a}) V0a) V1b)))))) \quad (39)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_{.27a}).((p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ (ap\ (c_2Eordinal_2EfromNat \\
& A_{.27a})\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))))\ V0a)) \Rightarrow ((ap\ (ap\ (c_2Eordinal_2Epolyform \\
& A_{.27a})\ V0a)\ (ap\ (c_2Eordinal_2EfromNat\ A_{.27a})\ c_2Enum_2E0)) = \\
& (c_2Elist_2ENIL\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& A_{.27a})\ (ty_2Eordinal_2Eordinal\ A_{.27a))))))
\end{aligned} \tag{40}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0a \in (ty_2Eordinal_2Eordinal \\
& A_{.27a}).(\forall V1x \in (ty_2Eordinal_2Eordinal\ A_{.27a}).((p\ (ap \\
& (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ (ap\ (c_2Eordinal_2EfromNat\ A_{.27a}) \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\
& V0a)) \Rightarrow (((ap\ (ap\ (c_2Eordinal_2Epolyform\ A_{.27a})\ V0a)\ V1x) = (c_2Elist_2ENIL \\
& (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal\ A_{.27a})\ (ty_2Eordinal_2Eordinal \\
& A_{.27a)))))) \Leftrightarrow (V1x = (ap\ (c_2Eordinal_2EfromNat\ A_{.27a})\ c_2Enum_2E0))))))
\end{aligned}$$