

thm_2Eordinal_2Epreimage_sup_thm
(TMbo1Yku8aLbT1inUP83xfyMTDdXAX6LDL1)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Eenum_2E_enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2E_enum \tag{1}$$

Let $ty_2Esum_2E_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2E_sum\ A0\ A1) \tag{2}$$

Let $ty_2Ewellorder_2E_wellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2E_wellorder\ A0) \tag{3}$$

Let $ty_2Eordinal_2E_ordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2E_ordinal\ A0) \tag{4}$$

Let $c_2Eordinal_2E_ordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2E_ordinal_REP_CLASS\ A_27a) (ty_2Ewellorder_2E_wellorder\ (ty_2Esum_2E_sum\ ty_2Eenum_2E_enum\ A_27a))) (ty_2Eordinal_2E_ordinal\ A_27a) \tag{5}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 5 We define $c_2Eordinal_2E_ordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2E_ordinal\ A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (7)$$

Definition 6 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 8 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2EF))$

Definition 9 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V2t))))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 10 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y))$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (10)$$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 13 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 14 We define $c_Ewellorder_Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 15 We define $c_Eset_relation_Erestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (12)$$

Definition 16 We define $c_Ewellorder_Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder\ A_27a)$

Definition 17 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40\ A_27a)\ P))))$

Definition 18 We define $c_Eset_relation_ERange$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 19 We define $c_Eset_relation_EDomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 20 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$

Definition 21 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2E_22\ A_27a)\ s\ t))$

Definition 22 We define $c_Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 23 We define $c_Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 24 We define $c_Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 25 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a). \lambda V1T2 \in (ty_2Eordinal_2Eordinal\ A_27a). (c_2Eordlt\ T1\ T2))$

Definition 26 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a). (\lambda V1s \in (ty_2Eordinal_2Eordinal\ A_27a). (c_2Epreds\ w\ s))$

Definition 27 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (ty_2Eordinal_2Eordinal\ A_27a). (c_2Eimage\ f\ s))$

Definition 28 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(2^{A_27a})}). (ap\ (c_2Epred_set_2E_28\ A_27a)\ P))$

Definition 29 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota. \lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (ap\ (c_2Eoleast\ P))$

Definition 30 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota. \lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). (c_2Esup\ ordset)$

Definition 31 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2ET)$.

Definition 32 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (ty_2Eordinal_2Eordinal\ A_27a). (c_2Einj\ f\ s))$

Definition 33 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0s1 \in (2^{A_27a}). \lambda V1s2 \in (2^{A_27b}). (c_2Ecardleq\ s1\ s2))$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. ((p V0t1) \Rightarrow (p V1t2)) \Rightarrow ((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (15)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A_27a. (p (ap V1Q V3x)))))) \quad (20)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (21)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A-27a}). (\forall V1a \in A_27a. ((\exists V2x \in A_27a. ((V2x = V1a) \wedge (p (ap\ V0P\ V2x)))) \Leftrightarrow (p (ap\ V0P\ V1a)))))) \quad (23)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0s \in (2^{A-27a}). (\forall V1t \in (2^{A-27b}). \\ (\forall V2u \in (2^{A-27c}). ((p (ap (ap (c_2Ecardinal_2Ecardleq \\ A_27a\ A_27b)\ V0s)\ V1t)) \wedge (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27b \\ A_27c)\ V1t)\ V2u))) \Rightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq\ A_27a\ A_27c) \\ V0s)\ V2u)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0f \in (A_27b^{A-27a}). (\forall V1s \in (2^{A-27a}). (p (ap (ap (\\ c_2Ecardinal_2Ecardleq\ A_27b\ A_27a) (ap (ap (c_2Epred_set_2EIMAGE \\ A_27a\ A_27b)\ V0f)\ V1s)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A_27a). (\forall V1w \in (ty_2Eordinal_2Eordinal\ A_27a). ((p (ap \\ (ap (c_2Ebool_2EIN (ty_2Eordinal_2Eordinal\ A_27a))\ V0x) (ap (\\ c_2Eordinal_2Epreds\ A_27a)\ V1w))) \Leftrightarrow (p (ap (ap (c_2Eordinal_2Eordlt \\ A_27a)\ V0x)\ V1w)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ord \in (ty_2Eordinal_2Eordinal \\ A_27a). (p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal \\ A_27a) (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)) (ap (c_2Eordinal_2Epreds \\ A_27a)\ V0ord)) (c_2Epred_set_2EUNIV (ty_2Esum_2Esum\ ty_2Enum_2Enum \\ A_27a)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\ ((p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal \\ A_27a) (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V0s) (c_2Epred_set_2EUNIV \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \Rightarrow (\forall V1a \in (ty_2Eordinal_2Eordinal \\ A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V1a) (ap (c_2Eordinal_2Esup \\ A_27a)\ V0s))) \Leftrightarrow (\exists V2b \in (ty_2Eordinal_2Eordinal\ A_27a). \\ ((p (ap (ap (c_2Ebool_2EIN (ty_2Eordinal_2Eordinal\ A_27a))\ V2b) \\ V0s)) \wedge (p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V1a)\ V2b)))))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & \quad A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & \quad (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \\ & \end{aligned} \tag{29}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{30}$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{31}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \\ & \end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{34}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))) \\ & \end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))) \\ & \end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \vee (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee (\neg(p\ V1q))) \wedge (((p\ V0p) \vee (\neg(p\ V2r))) \wedge \\ & \quad ((p\ V1q) \vee ((p\ V2r) \vee (\neg(p\ V0p)))))))) \\ & \end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee \neg(p V2r))) \wedge (\neg(p V1q)) \vee ((p V2r) \vee \neg(p V0p)))))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow \neg(p V1q)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (\neg(p V1q)) \vee \neg(p V0p)))))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow \neg(p V1q))) \quad (41)$$

Theorem 1

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal A_27a)^{(ty_2Eordinal_2Eordinal A_27a)}). (\forall V1a \in (ty_2Eordinal_2Eordinal A_27a). (\forall V2b \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2b) (ap (c_2Eordinal_2Esup A_27a) (ap (ap (c_2Epred_set_2EIMAGE (ty_2Eordinal_2Eordinal A_27a) (ty_2Eordinal_2Eordinal A_27a)) V0f) (ap (c_2Eordinal_2Epreds A_27a) V1a)))))) \Leftrightarrow (\exists V3d \in (ty_2Eordinal_2Eordinal A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt A_27a) V3d) V1a)) \wedge (p (ap (ap (c_2Eordinal_2Eordlt A_27a) V2b) (ap V0f V3d))))))))))$$