

thm_2Eordinal_2Epreds__downward__closed (TM- LoDuXvtxsFwkgGf9E2i2QwGVLqKpC5wQw)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \tag{4}$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) (ty_2Ewellorder_2Ewellorder\ A_27a)) \tag{5}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E_2C$

Definition 9 We define `c_2Ebool_2EIN` to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let `c_2Epair_2EFST` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

Definition 10 We define `c_2Epair_2EUNCURRY` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let `c_2Epred_set_2EGSPEC` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

Definition 11 We define `c_2Eset_relation_2Estrict` to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 12 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A) p)$ of type $\iota \Rightarrow \iota$.

Definition 13 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define `c_2Eset_relation_2Erange` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 15 We define `c_2Eset_relation_2Edomain` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 16 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Definition 17 We define `c_2Epred_set_2EUNION` to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap (c_2Epred_set_2EUNION$

Definition 18 We define `c_2Ewellorder_2EelsOf` to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 19 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Eorderiso$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})}) \quad (11)$$

Definition 20 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota. \lambda V0r \in (ty_2Ewellorder_2Ewellorder$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS\ A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (12)$$

Definition 21 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 22 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 23 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a))})}) \quad (13)$$

Definition 24 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder$

Definition 25 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 26 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a).$

Definition 27 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a).$

Definition 28 We define $c_2Eordinal_2Edownward_closed$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal$

Definition 29 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x)$

Definition 30 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a})$

Definition 31 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 32 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0j$

Definition 33 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R1 \in ((2^{A_27a})^{A_27a})$

Definition 34 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a})$

Definition 35 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in ((A_27b^{A_27a})^{A_27a}).(\lambda V1x$

Definition 36 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(c_2Ecombin_2EW A_27a A_27b$

Definition 37 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota.(\lambda V0p \in (2^{A_27a}).(\lambda V1m \in (2^{A_27a}).$

Definition 38 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota.\lambda V0E \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}).(\forall V1g \in (A_27b^{A_27a}).((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_27a.((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ & V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \end{aligned} \tag{21}$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (22)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2}Ecombin_{.2EI} A_{.27a}) V0x) = V0x)) \quad (23)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} (ty_{.2}Ewellorder_{.2Ewellorder} (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (ty_{.2}Eordinal_{.2Eordinal} A_{.27a})) (c_{.2}Ewellorder_{.2Eorderiso} (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (ty_{.2}Esum_{.2Esum} ty_{.2}Enum_{.2Enum} A_{.27a})) (c_{.2}Eordinal_{.2Eordinal}_{.2ABS} A_{.27a})) (c_{.2}Eordinal_{.2Eordinal}_{.2REP} A_{.27a}))) \quad (24)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in (ty_{.2}Eordinal_{.2Eordinal} A_{.27a}).(\forall V1w \in (ty_{.2}Eordinal_{.2Eordinal} A_{.27a}).((p (ap (ap (c_{.2}Ebool_{.2EIN} (ty_{.2}Eordinal_{.2Eordinal} A_{.27a})) V0x) (ap (c_{.2}Eordinal_{.2Eprede} A_{.27a}) V1w))) \Leftrightarrow (p (ap (ap (c_{.2}Eordinal_{.2Eordlt} A_{.27a}) V0x) V1w)))))) \quad (25)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27a} A_{.27a}) (c_{.2}Emin_{.2E_{.3D}} A_{.27a})) (c_{.2}Ecombin_{.2EI} A_{.27a})) (c_{.2}Ecombin_{.2EI} A_{.27a}))) \quad (26)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow \forall A_{.27d}.nonempty A_{.27d} \Rightarrow (\forall V0R1 \in (2^{A_{.27a}A_{.27a}}).(\forall V1abs1 \in (A_{.27c}^{A_{.27a}}).(\forall V2rep1 \in (A_{.27a}^{A_{.27c}}).((p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27a} A_{.27c}) V0R1) V1abs1) V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_{.27b}A_{.27b}}).(\forall V4abs2 \in (A_{.27d}^{A_{.27b}}).(\forall V5rep2 \in (A_{.27b}^{A_{.27d}}).((p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} A_{.27b} A_{.27d}) V3R2) V4abs2) V5rep2))) \Rightarrow (p (ap (ap (ap (c_{.2}Equotient_{.2EQUOTIENT} (A_{.27b}^{A_{.27a}}) (A_{.27d}^{A_{.27c}})) (ap (ap (c_{.2}Equotient_{.2E_{.3D}_{.3D}_{.3D}_{.3E}} A_{.27a} A_{.27b}) V0R1) V3R2)) (ap (ap (c_{.2}Equotient_{.2E_{.2D}_{.2D}_{.3E}} A_{.27c} A_{.27b} A_{.27a} A_{.27d}) V2rep1) V4abs2)) (ap (ap (c_{.2}Equotient_{.2E_{.2D}_{.2D}_{.3E}} A_{.27a} A_{.27d} A_{.27c} A_{.27b}) V1abs1) V5rep2)))))))))) \quad (27)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{28}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep)) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2))\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))))
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a.(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y)))))))))))))
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P))))))
\end{aligned} \tag{33}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\
& \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\
& \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge ((p \ V0p) \vee (\neg(p \ V2r)))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge (\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{44}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\
& A_27a).(p \ (ap \ (ap \ (c_2Ewellorder_2Eorderiso \ A_27a \ A_27a) \ V0w) \\
& V0w)))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0w1 \in (ty_2Ewellorder_2Ewellorder \ A_27a). (\forall V1w2 \in \\
& (ty_2Ewellorder_2Ewellorder \ A_27b). ((p \ (ap \ (ap \ (c_2Ewellorder_2Eorderiso \\
& A_27a \ A_27b) \ V0w1) \ V1w2)) \Rightarrow (p \ (ap \ (ap \ (c_2Ewellorder_2Eorderiso \\
& A_27b \ A_27a) \ V1w2) \ V0w1))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A.27a). \\
& (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A.27b). (\forall V2w3 \in \\
& (ty_2Ewellorder_2Ewellorder\ A.27c). (((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& A.27a\ A.27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& A.27b\ A.27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& A.27a\ A.27c)\ V0w1)\ V2w3))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A.27a). \\
& (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A.27b). (\forall V2w3 \in \\
& (ty_2Ewellorder_2Ewellorder\ A.27c). (((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\
& A.27a\ A.27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\
& A.27b\ A.27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\
& A.27a\ A.27c)\ V0w1)\ V2w3))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0x0 \in (\\
& ty_2Ewellorder_2Ewellorder\ A.27a). (\forall V1y0 \in (ty_2Ewellorder_2Ewellorder \\
& A.27b). (\forall V2a0 \in (ty_2Ewellorder_2Ewellorder\ A.27c). (\\
& \forall V3b0 \in (ty_2Ewellorder_2Ewellorder\ A.27d). (((p\ (ap\ (ap \\
& (c_2Ewellorder_2Eorderiso\ A.27a\ A.27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap \\
& (c_2Ewellorder_2Eorderiso\ A.27c\ A.27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap \\
& (ap\ (c_2Ewellorder_2Eorderlt\ A.27a\ A.27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap \\
& (ap\ (c_2Ewellorder_2Eorderlt\ A.27b\ A.27d)\ V1y0)\ V3b0))))))
\end{aligned} \tag{50}$$

Theorem 1

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2Eordinal_2Eordinal \\
A.27a). (p\ (ap\ (c_2Eordinal_2Edownward_closed\ A.27a)\ (ap\ (c_2Eordinal_2Epreds \\
A.27a)\ V0w))))$$