

thm_2Eordinal_2Epreds__inj__univ
(TMEhfFyUtFNnumnX3HtSa7SJpy23nkYBkdvd)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_7E` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}})) (\lambda V1f \in 2.V1f) V0P)))$

Definition 4 We define `c_2Ebool_2E_7E` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 7 We define `c_2Ebool_2E_7E` to be $\lambda A_{27a} : \iota.(\lambda V0x \in A_{27a}.(\lambda V1f \in (2^{A_{27a}}).(ap V1f V0x)))$

Definition 8 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2)))$

Definition 9 We define `c_2Epred__set_2EINJ` to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0f \in (A_{27b}^{A_{27a}}).\lambda V1s \in (2^{A_{27b}})$

Definition 10 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) then (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40) V0P)))$

Definition 12 We define `c_2Ecardinal_2Ecardleq` to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0s1 \in (2^{A_{27a}}).\lambda V1s2 \in (2^{A_{27b}})$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (3)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (4)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (5)$$

Definition 13 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ x\ y)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (10)$$

Definition 15 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 16 We define $c_Eset_relation_Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Definition 17 We define $c_Eset_relation_ERange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 18 We define $c_Eset_relation_EDomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 19 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in 2))))$

Definition 20 We define $c_Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_Ebool_2E_21\ 2)\ (\lambda V2t \in 2))$

Definition 21 We define $c_Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 22 We define $c_Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS \\ & A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Eenum_2Eenum\ A_27a))})}) \end{aligned} \quad (12)$$

Definition 23 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 24 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 25 We define $c_Eset_relation_ERrestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (13)$$

Definition 26 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 27 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 28 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 29 We define $c_2Eordinal_2EallOrds$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewellorder_2Ewellorder_ABS\ (ty_2Ewellorder_2Ewellorder\ A_27a)))$

Definition 30 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 31 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_2E_2ET)$

Definition 32 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27a^{A_27b})$

Definition 33 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).\lambda V1s \in (A_27a^{A_27b})$

Definition 34 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (A_27a^{A_27b})$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (16) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (17) \end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p\ V0t)))))) \quad (20) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\ & (ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\ & A_27a))\ (ty_2Eordinal_2Eordinal\ A_27a))\ (c_2Ewellorder_2Eorderiso \\ & (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\ & A_27a))\ (c_2Eordinal_2Eordinal_ABS\ A_27a))\ (c_2Eordinal_2Eordinal_REP \\ & A_27a))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a)).(p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a)\ (ty_2Eordinal_2Eordinal \\ A.27a))\ V0w)\ (ap\ (ap\ (c_2Ewellorder_2Ewobound\ (ty_2Eordinal_2Eordinal \\ A.27a))\ (ap\ (c_2Eordinal_2Eordinal_ABS\ A.27a)\ V0w))\ (c_2Eordinal_2EallOrds \\ A.27a)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0ord \in (ty_2Eordinal_2Eordinal \\ A.27a)).((ap\ (c_2Eordinal_2Epreds\ A.27a)\ V0ord) = (ap\ (c_2Ewellorder_2EelsOf \\ (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ (ap\ (c_2Ewellorder_2Ewobound \\ (ty_2Eordinal_2Eordinal\ A.27a))\ V0ord)\ (c_2Eordinal_2EallOrds \\ A.27a)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(p\ (ap\ (ap\ (c_2Ebool_2EIN \\ A.27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A.27a)))) \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A.27a)).(\forall V1w2 \in \\ (ty_2Ewellorder_2Ewellorder\ A.27b)).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ A.27a\ A.27b)\ V0w1)\ V1w2)) \Leftrightarrow (\exists V2f \in (A.27b^{A.27a}).((p\ (ap\ (\\ ap\ (ap\ (c_2Epred_set_2EBIJ\ A.27a\ A.27b)\ V2f)\ (ap\ (c_2Ewellorder_2EelsOf \\ A.27a)\ V0w1))\ (ap\ (c_2Ewellorder_2EelsOf\ A.27b)\ V1w2))) \wedge (\forall V3x \in \\ A.27a.(\forall V4y \in A.27a.((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod \\ A.27a\ A.27a))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27a\ A.27a)\ V3x)\ V4y))\ (ap \\ (c_2Eset_relation_2Estrict\ A.27a))\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\ A.27a)\ V0w1)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A.27b \\ A.27b))\ (ap\ (ap\ (c_2Epair_2E_2C\ A.27b\ A.27b)\ (ap\ V2f\ V3x))\ (ap\ V2f \\ V4y)))\ (ap\ (c_2Eset_relation_2Estrict\ A.27b)\ (ap\ (c_2Ewellorder_2Ewellorder_REP \\ A.27b)\ V1w2)))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ A.27a)).(p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A.27a\ A.27a)\ V0w) \\ V0w))) \quad (28)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). (\forall V1w2 \in \\
& (ty_2Ewellorder_2Ewellorder\ A_27b). ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& \quad A_27b\ A_27a)\ V1w2)\ V0w1))))))
\end{aligned} \tag{29}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0ord \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a). (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal \\
& A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ (ap\ (c_2Eordinal_2Epreds \\
& \quad A_27a)\ V0ord))\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\
& \quad A_27a))))))
\end{aligned}$$