

thm_2Eordinal_2Epreds_surj (TM- JAB37mKM8Dch7i7zRyqAoXZRuLtGbEpeb)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 4 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2.V0t))$.

Let `ty_2Eordinal_2Eordinal` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eordinal_2Eordinal } A0) \quad (1)$$

Definition 5 We define `c_2Ebool_2EIN` to be $\lambda A. \lambda a : \iota. (\lambda V0x \in A. \lambda a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f V0x)))$

Let `ty_2Eenum_2Eenum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Eenum_2Eenum} \quad (2)$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 A1) \quad (3)$$

Let `ty_2Ewellorder_2Ewellorder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ewellorder_2Ewellorder } A0) \quad (4)$$

Let `c_2Eordinal_2Eordinal__REP__CLASS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$A. \lambda a \in ((2^{(\text{ty_2Ewellorder_2Ewellorder } (\text{ty_2Esum_2Esum } \text{ty_2Eenum_2Eenum } A. \lambda a))}) (\text{ty_2Eordinal_2Eordinal } A. \lambda a)) \quad (5)$$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A. \lambda p \in 2^A. \text{if } (\exists x \in A. p (\text{ap } P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p (\text{ap } P x)))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)}) \quad (7)$$

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. (ap (c_2Ebool_2E_7E) V2t) V1t2) V0t1)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b.(ap (c_2Epair_2E_2C) V0x V1y)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (10)$$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27b})^{A_27a}$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \quad (11)$$

Definition 13 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 14 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 15 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (12)$$

Definition 16 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder\ A_27a)$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E40\ A_27a)\ P)))$

Definition 18 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 19 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E_3F)\ s\ t)$

Definition 22 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 23 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 24 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 25 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 26 We define $c_2Eordinal_2Edownward_closed$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$

Definition 27 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 28 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2ET)$

Definition 29 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E_2EF)$

Definition 30 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_2ET)\ s)$

Definition 31 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Ebool_2E_2EF)\ s\ t)$

Definition 32 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1x \in A_27a. (ap\ (c_2Ebool_2E_2EF)\ s)$

Definition 33 We define $c_2Epred_set_2ESURJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_2ET)\ s)$

Definition 34 We define $c_2Epred_set_2EINJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_2EF)\ s)$

Definition 35 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0f \in (A_27b^{A_27a}). \lambda V1s \in (2^{A_27a}). (ap\ (c_2Ebool_2E_2ET)\ s)$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (14)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\
& A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& p \ V0t)))))) \quad (16)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (17)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \quad (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (p \ (ap \ (ap \ (ap \ (c.2Epred_set.2EBIJ \\
& (ty.2Eordinal.2Eordinal \ A.27a) \ (2^{(ty.2Eordinal.2Eordinal \ A.27a)})) \\
& (c.2Eordinal.2Epreds \ A.27a)) \ (c.2Epred_set.2EUNIV \ (ty.2Eordinal.2Eordinal \\
& A.27a))) \ (ap \ (ap \ (c.2Epred_set.2EDELETE \ (2^{(ty.2Eordinal.2Eordinal \ A.27a)})) \\
& (c.2Eordinal.2Edownward_closed \ A.27a)) \ (c.2Epred_set.2EUNIV \\
& (ty.2Eordinal.2Eordinal \ A.27a)))))) \quad (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1x \in \\
& A.27a.((p \ (ap \ (ap \ (c.2Ebool.2EIN \ A.27a) \ V1x) \ V0P)) \Leftrightarrow (p \ (ap \ V0P \ V1x)))))) \quad (20)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V0x)\ (c.2Epred_set.2EUNIV\ A.27a)))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1x \in A.27a. (\forall V2y \in A.27a. ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ (ap\ (ap\ (c.2Epred_set.2EDELETE\ A.27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap\ (c.2Ebool.2EIN\ A.27a)\ V1x)\ V0s)) \wedge (\neg(V1x = V2y))))))) \quad (22)$$

Theorem 1

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in (2^{(ty.2Eordinal.2Eordinal\ A.27a)}). (((p\ (ap\ (c.2Eordinal.2Edownward_closed\ A.27a)\ V0x)) \wedge (\neg(V0x = (c.2Epred_set.2EUNIV\ (ty.2Eordinal.2Eordinal\ A.27a)))))) \Rightarrow (\exists V1y \in (ty.2Eordinal.2Eordinal\ A.27a). ((ap\ (c.2Eordinal.2Epreds\ A.27a)\ V1y) = V0x))))$$