thm_2Eordinal_2Epreds__wobound (TMQ6BcmviM7thb4rCCVmefADoGi6w1v6heu)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \tag{3}$$

Let $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \tag{4}$$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \tag{5}$$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_2$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{7}$$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{8}$$

Definition 10 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Definition 11 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{9}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{10}$$

Definition 12 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

Definition 13 *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})$

Definition 14 *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

Definition 15 *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a:\iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (12)$$

Definition 16 *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a:\iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a:\iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 18 *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

Definition 19 *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

Definition 20 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 21 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a:\iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 22 *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A$

Definition 23 *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1 \in (ty\_2Ewellorder\_2$

Definition 24 *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

Definition 25 *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a:\iota.\lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

Definition 26 *We define* c_2Eordinal_2EallOrds *to be* $\lambda A\_27a:\iota.(ap\ (c\_2Ewellorder\_2Ewellorder\_\_ABS\ (t$

Definition 27 *We define* c_2Eordinal_2Epreds *to be* $\lambda A\_27a:\iota.\lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).($

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1y \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a)))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0x)\ V1y))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (\mathsf{c\_2Eordinal\_2EallOrds}\ A\_27a))))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0x)\ V1y))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0x)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1w))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0x)\ V1w))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}).(\forall V1t \in (2^{A\_27a}).((V0s = V1t) \Leftrightarrow (\forall V2x \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V2x)\ V1t))))))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in ((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}).(\forall V1v \in A\_27a.((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V1v)\ (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V1v)\ \mathsf{c\_2Ebool\_2ET}) = (ap\ V0f\ V2x)))))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1w \in (ty\_2Ewellorder\_2Ewellorder\ A\_27a).((ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ A\_27a)\ V0x)\ V1w)) = (ap\ (c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27a)\ (\lambda V2y \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ 2)\ V2y)\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27a)\ V2y)\ V0x))\ (ap\ (\mathsf{c\_2Eset\_\_relation\_2Estrict}\ A\_27a)\ (ap\ (c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a)\ V1w))))))))) \quad (23)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ord \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V0ord) = (ap\ (\mathsf{c\_2Ewellorder\_2EelsOf}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ (ap\ (ap\ (\mathsf{c\_2Ewellorder\_2Ewobound}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0ord)\ (\mathsf{c\_2Eordinal\_2EallOrds}\ A\_27a)))))$$