

thm_2Eordinal_2Eubsup_thm (TMWzrPpayY- WhQ3RLXxeNXu5XQfn7Eg1nPdR)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{3}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \tag{4}$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP\ A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})(ty_2Ewellorder_2Ewellorder\ A_27a)) \tag{5}$$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 5 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21\ 2)) (\lambda V0t \in 2.V0t)$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (6)$$

Definition 9 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod$

Definition 10 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).ap V1f V0x))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (7)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (8)$$

Definition 11 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (9)$$

Definition 12 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27a)})$

Definition 13 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).ap V0P (ap (c_2Emin_2E_40$

Definition 14 We define $c_2Eset_relation_2Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 15 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A_27a A_27b)})$

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2)))$

Definition 17 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).ap (c_2Emin_2E_40$

Definition 18 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A_27a)$

Definition 19 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Eorderiso$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (10)$$

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})}) \quad (11)$$

Definition 20 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ewellorder_2Ewellorder$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_REP_CLASS\ A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (12)$$

Definition 21 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 22 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$

Definition 23 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (13)$$

Definition 24 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder$

Definition 25 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder$

Definition 26 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 27 We define $c_2Eordinal_2Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a)$

Definition 28 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$

Definition 29 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 30 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})})$

Definition 31 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$

Definition 32 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 33 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. (\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 34 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Definition 35 We define $c_2Equotient_2E_2D_2D_3E$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda A_27d : \iota. \lambda V0f \in ((2^{A_27a})^{A_27b})$

Definition 36 We define $c_2Equotient_2E_3D_3D_3D_3E$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R1 \in ((2^{A_27a})^{A_27b})$

Definition 37 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27b})$

Definition 38 We define $c_2Ecombin_2EW$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0f \in ((A_27b^{A_27a})^{A_27a}). (\lambda V1x \in A_27a))$

Definition 39 We define $c_2Equotient_2Erespects$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (c_2Ecombin_2EW A_27a A_27a)$

Definition 40 We define $c_2Ebool_2ERES_FORALL$ to be $\lambda A_27a : \iota. (\lambda V0p \in (2^{A_27a}). (\lambda V1m \in (2^{A_27a}). \lambda V2x \in A_27a))$

Definition 41 We define $c_2Equotient_2EEQUIV$ to be $\lambda A_27a : \iota. \lambda V0E \in ((2^{A_27a})^{A_27a}). (ap (c_2Ebool_2ERES_FORALL A_27a V0E))$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow \neg(p V0t))))) \tag{16}$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \tag{17}$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{18}$$

Assume the following.

$$\forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap V0f V2x) = (ap V1g V2x))))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(p V0t))))) \tag{20}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\forall V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p (ap V0P V2x)))))) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A.27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A.27a.(p (ap V1Q V3x))))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A)) \vee (\neg(p V1B)))))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2Ecombin}_{.2EI} A_{.27a}) V0x) = V0x)) \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (p (ap (ap (ap (c_{.2Equotient}_{.2EQUOTIENT} (ty_{.2Ewellorder}_{.2Ewellorder} (ty_{.2Esum}_{.2Esum} ty_{.2Enum}_{.2Enum} A_{.27a})) (ty_{.2Eordinal}_{.2Eordinal} A_{.27a})) (c_{.2Ewellorder}_{.2Eorderiso} (ty_{.2Esum}_{.2Esum} ty_{.2Enum}_{.2Enum} A_{.27a})) (ty_{.2Esum}_{.2Esum} ty_{.2Enum}_{.2Enum} A_{.27a})) (c_{.2Eordinal}_{.2Eordinal}_{.2ABS} A_{.27a})) (c_{.2Eordinal}_{.2Eordinal}_{.2REP} A_{.27a}))) \quad (37)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in (ty_{.2Eordinal}_{.2Eordinal} A_{.27a}).(\forall V1w \in (ty_{.2Eordinal}_{.2Eordinal} A_{.27a}).((p (ap (ap (c_{.2Ebool}_{.2EIN} (ty_{.2Eordinal}_{.2Eordinal} A_{.27a})) V0x) (ap (c_{.2Eordinal}_{.2Eprede} A_{.27a}) V1w)) \Leftrightarrow (p (ap (ap (c_{.2Eordinal}_{.2Eordlt} A_{.27a}) V0x) V1w)))))) \quad (38)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a). (\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a). ((\neg(p (\\ ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V0b)\ V1a))) \Leftrightarrow ((p (ap (ap (c_2Eordinal_2Eordlt \\ A_27a)\ V1a)\ V0b)) \vee (V1a = V0b)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0Q \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}), \\ (\forall V1P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). ((\exists V2a \in \\ (ty_2Eordinal_2Eordinal\ A_27a). (p (ap\ V1P\ V2a))) \wedge (\forall V3a \in \\ (ty_2Eordinal_2Eordinal\ A_27a). ((\forall V4b \in (ty_2Eordinal_2Eordinal \\ A_27a). ((p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V4b)\ V3a)) \Rightarrow (\neg \\ p (ap\ V1P\ V4b)))))) \wedge (p (ap\ V1P\ V3a))) \Rightarrow (p (ap\ V0Q\ V3a)))))) \Rightarrow (p (ap\ V0Q \\ (ap (c_2Eordinal_2Eoleast\ A_27a)\ V1P)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ ((p (ap (ap (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap (ap (c_2Epred_set_2EIMAGE \\ A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ (p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1sos \in \\ (2^{(2^{A_27a})}). ((p (ap (ap (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap (c_2Epred_set_2EBIGUNION \\ A_27a)\ V1sos))) \Leftrightarrow (\exists V2s \in (2^{A_27a}). ((p (ap (ap (c_2Ebool_2EIN \\ A_27a)\ V0x)\ V2s)) \wedge (p (ap (ap (c_2Ebool_2EIN\ (2^{A_27a})\ V2s)\ V1sos)))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (p (ap (ap (ap (c_2Equotient_2EQUOTIENT \\ A_27a\ A_27a)\ (c_2Emin_2E_3D\ A_27a))\ (c_2Ecombin_2EI\ A_27a))\ (\\ c_2Ecombin_2EI\ A_27a))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad (A_27b^{A_27a})\ (A_27d^{A_27c}))\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E \\
& \quad A_27a\ A_27b)\ V0R1)\ V3R2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E\ A_27c \\
& \quad A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2))\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27a\ A_27d\ A_27c\ A_27b)\ V1abs1)\ V5rep2))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1))) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2))) \Rightarrow (\forall V6f \in (A_27d^{A_27c}). \\
& \quad ((\lambda V7x \in A_27c.(ap\ V6f\ V7x)) = (ap\ (ap\ (ap\ (c_2Equotient_2E_2D_2D_3E \\
& \quad A_27c\ A_27b\ A_27a\ A_27d)\ V2rep1)\ V4abs2)\ (\lambda V8x \in A_27a.(ap\ V5rep2 \\
& \quad (ap\ V6f\ (ap\ V1abs1\ V8x))))))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0REL \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0REL)\ V1abs)\ V2rep))) \Rightarrow (\forall V3x1 \in A_27a.(\forall V4x2 \in \\
& \quad A_27a.((p\ (ap\ (ap\ V0REL\ V3x1)\ V4x2)) \Rightarrow (p\ (ap\ (ap\ V0REL\ V3x1)\ (ap\ V2rep \\
& \quad (ap\ V1abs\ V4x2))))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep))) \Rightarrow (\forall V3f \in (2^{A_27b}).((p\ (\\
& \quad ap\ (c_2Ebool_2E_21\ A_27b)\ V3f)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2ERespects\ A_27a\ 2)\ V0R))\ (ap\ (ap\ (ap \\
& \quad (c_2Equotient_2E_2D_2D_3E\ A_27a\ 2\ A_27b\ 2)\ V1abs)\ (c_2Ecombin_2EI \\
& \quad 2))\ V3f))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0R \in ((2^{A_27a})^{A_27a}).(\forall V1abs \in (A_27b^{A_27a}). \\
& \quad (\forall V2rep \in (A_27a^{A_27b}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27a\ A_27b)\ V0R)\ V1abs)\ V2rep)) \Rightarrow (\forall V3f \in (2^{A_27a}).(\forall V4g \in \\
& \quad (2^{A_27a}).((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad 2)\ V0R)\ (c_2Emin_2E_3D\ 2)\ V3f)\ V4g)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL \\
& \quad A_27a)\ (ap\ (c_2Equotient_2Erespects\ A_27a\ 2)\ V0R))\ V3f)) \Leftrightarrow (p\ (\\
& \quad ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0R))\ V4g))))))))) \\
& \hspace{15em} (48)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& \quad nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0R1 \in (\\
& \quad (2^{A_27a})^{A_27a}).(\forall V1abs1 \in (A_27c^{A_27a}).(\forall V2rep1 \in \\
& \quad (A_27a^{A_27c}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ A_27a\ A_27c) \\
& \quad V0R1)\ V1abs1)\ V2rep1)) \Rightarrow (\forall V3R2 \in ((2^{A_27b})^{A_27b}).(\forall V4abs2 \in \\
& \quad (A_27d^{A_27b}).(\forall V5rep2 \in (A_27b^{A_27d}).((p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT \\
& \quad A_27b\ A_27d)\ V3R2)\ V4abs2)\ V5rep2)) \Rightarrow (\forall V6f \in (A_27b^{A_27a}). \\
& \quad (\forall V7g \in (A_27b^{A_27a}).(\forall V8x \in A_27a).(\forall V9y \in \\
& \quad A_27a.(((p\ (ap\ (ap\ (ap\ (ap\ (c_2Equotient_2E_3D_3D_3D_3E\ A_27a \\
& \quad A_27b)\ V0R1)\ V3R2)\ V6f)\ V7g)) \wedge (p\ (ap\ (ap\ V0R1\ V8x)\ V9y))) \Rightarrow (p\ (ap\ (\\
& \quad ap\ V3R2\ (ap\ V6f\ V8x))\ (ap\ V7g\ V9y))))))))) \\
& \hspace{15em} (49)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0E \in ((2^{A_27a})^{A_27a}). \\
& \quad (\forall V1P \in (2^{A_27a}).((p\ (ap\ (c_2Equotient_2EEQUIV\ A_27a) \\
& \quad V0E)) \Rightarrow ((p\ (ap\ (ap\ (c_2Ebool_2ERES_FORALL\ A_27a)\ (ap\ (c_2Equotient_2Erespects \\
& \quad A_27a\ 2)\ V0E))\ V1P)) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ A_27a)\ V1P)))))) \\
& \hspace{15em} (50)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \hspace{10em} (51)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \hspace{10em} (52)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (53)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\
& \hspace{15em} (54)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (62)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ & A_27a).(p (ap (ap (c_2Ewellorder_2Eorderiso A_27a A_27a) V0w) \\ & V0w))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\ & (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27b\ A_27a)\ V1w2)\ V0w1)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). \\ & \quad (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2w3 \in \\ & (ty_2Ewellorder_2Ewellorder\ A_27c).(((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27b\ A_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\ & \quad A_27a\ A_27c)\ V0w1)\ V2w3)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder \\ & \quad A_27a).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27a)\ V0w) \\ & \quad V0w)) \Leftrightarrow False)) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a). \\ & \quad (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2w3 \in \\ & (ty_2Ewellorder_2Ewellorder\ A_27c).(((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\ & \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\ & \quad A_27b\ A_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\ & \quad A_27a\ A_27c)\ V0w1)\ V2w3)))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0x0 \in (\\ & ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1y0 \in (ty_2Ewellorder_2Ewellorder \\ & \quad A_27b).(\forall V2a0 \in (ty_2Ewellorder_2Ewellorder\ A_27c). \\ & \quad (\forall V3b0 \in (ty_2Ewellorder_2Ewellorder\ A_27d).(((p\ (ap\ (ap \\ & \quad (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap \\ & \quad (c_2Ewellorder_2Eorderiso\ A_27c\ A_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap \\ & \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap \\ & \quad (ap\ (c_2Ewellorder_2Eorderlt\ A_27b\ A_27d)\ V1y0)\ V3b0)))))) \end{aligned} \quad (68)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}. \text{nonempty } A_{.27a} \Rightarrow & (\forall V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_{.27a})}). \\ & (\forall V1b \in (ty_2Eordinal_2Eordinal\ A_{.27a}). ((\forall V2a \in \\ (ty_2Eordinal_2Eordinal\ A_{.27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal \\ A_{.27a})\ V2a)\ V0s)) \Rightarrow & (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ V2a) \\ & V1b)))) \Rightarrow (\forall V3c \in (ty_2Eordinal_2Eordinal\ A_{.27a}). ((p\ (ap \\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ V3c)\ (ap\ (c_2Eordinal_2Esup\ A_{.27a}) \\ V0s))) \Leftrightarrow & (\exists V4d \in (ty_2Eordinal_2Eordinal\ A_{.27a}). ((p\ (ap \\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal\ A_{.27a})\ V4d)\ V0s)) \wedge \\ & (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A_{.27a})\ V3c)\ V4d)))))))))) \end{aligned}$$