

thm_2Eordinal_2Euniv__cord__uncountable
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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$.
Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Definition 3 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda V1x \in A.c_2Ebool_2E_2ET)$.

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 5 We define $c_2Ebool_2E_2EIN$ to be $\lambda A.\lambda a : \iota.(\lambda V0x \in A.\lambda V1f \in (2^{A-27a}).(ap V1f V0x))$

Definition 6 We define $c_2Ebool_2E_2E21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 7 We define $c_2Ebool_2E_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_2E21 2) (\lambda V2t \in 2$

Definition 8 We define $c_2Epred_set_2EINJ$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0f \in (A.\lambda V1s \in (2^{A-$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_2E3F$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 11 We define $c_2Epred_set_2Ecountable$ to be $\lambda A.\lambda a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E_2E3F$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{2}$$

Definition 12 We define $c_2Epred_set_2ESURJ$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.\lambda V0f \in (A.\lambda V1s \in ($

Definition 13 We define $c_2Epred_set_2EBIJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^A$

Definition 14 We define $c_2Ecardinal_2Ecardeq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^A$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (3)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (4)$$

Definition 15 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^A$

Definition 16 We define c_2Ebool_2E2E to be $(ap\ (c_2Ebool_2E2E)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Ebool_2E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E2E)\ (\lambda V2t \in 2.V2t)$

Definition 18 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E2E$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (8)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (11)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (p (ap (ap (c_2Ecardinal_2Ecardeq A_27a A_27a) V0s) V0s))) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow \forall A_27b. \text{nonempty } A_27b \Rightarrow \forall A_27c. \\ \text{nonempty } A_27c \Rightarrow \forall A_27d. \text{nonempty } A_27d \Rightarrow (\forall V0s1 \in (\\ 2^{A_27a}). (\forall V1s2 \in (2^{A_27b}). (\forall V2t1 \in (2^{A_27c}). \\ (\forall V3t2 \in (2^{A_27d}). (((p (ap (ap (c_2Ecardinal_2Ecardeq \\ A_27a A_27b) V0s1) V1s2)) \wedge (p (ap (ap (c_2Ecardinal_2Ecardeq A_27c \\ A_27d) V2t1) V3t2)))) \Rightarrow ((p (ap (ap (c_2Ecardinal_2Ecardleq A_27a \\ A_27c) V0s1) V2t1)) \Leftrightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq A_27b A_27d) \\ V1s2) V3t2)))))))))) \end{aligned} \quad (13)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). ((p (ap (c_2Epred_set_2Ecountable A_27a) V0s)) \Leftrightarrow (p (ap (ap (c_2Ecardinal_2Ecardleq A_27a ty_2Enum_2Enum) V0s) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))))) \quad (14)$$

Assume the following.

$$(p (ap (ap (c_2Ecardinal_2Ecardeq (ty_2Esum_2Esum ty_2Enum_2Enum ty_2Eone_2Eone) ty_2Enum_2Enum) (c_2Epred_set_2EUNIV (ty_2Esum_2Esum ty_2Enum_2Enum ty_2Eone_2Eone))) (c_2Epred_set_2EUNIV ty_2Enum_2Enum)))) \quad (15)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\neg (p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal A_27a) (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)) (c_2Epred_set_2EUNIV (ty_2Eordinal_2Eordinal A_27a))) (c_2Epred_set_2EUNIV (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge ((p V0p) \vee ((\neg(\\ & p V2r)) \vee (\neg(p V1q)))))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (24) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (25) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (26) \end{aligned}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (28)$$

Theorem 1

$$\begin{aligned} & (\neg(p (ap (c.2Epred_set.2Ecountable (ty.2Eordinal.2Eordinal \\ & ty.2Eone.2Eone)) (c.2Epred_set.2EUNIV (ty.2Eordinal.2Eordinal \\ & ty.2Eone.2Eone)))))) \end{aligned}$$