

thm_2Eordinal_2Euniv__ord__greater__cardinal
 (TMXDxSx-
 AAV8btNCtLdMMs9TpB3eoBScDi2k)

October 26, 2020

Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ (ap } P \ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type $\iota.$

Definition 5 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 8 We define `c_2Epred__set_2EINJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27a}).$

Definition 9 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Definition 10 We define `c_2Ecardinal_2Ecardleq` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0s1 \in (2^{A-27a}). \lambda V1s2 \in (2^{A-27a}).$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 \ A1) \tag{2}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (3)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (4)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (5)$$

Definition 11 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (7)$$

Definition 12 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 13 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21\ 2))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 14 We define c_2Epair_2E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Emin_2E3D_3D_3E\ V0x\ V1y)\ c_2Ebool_2E7E)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (10)$$

Definition 15 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$.
Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \tag{11}$$

Definition 16 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Definition 17 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 18 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & \quad A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \tag{12}$$

Definition 19 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota. \lambda V0x \in A_27a. \lambda V1w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 20 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 21 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 22 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ t2))\ t1))))$.

Definition 23 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap\ (c_2Epred_set_2EUNION\ s)\ t))$.

Definition 24 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 25 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 26 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 27 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota. \lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$.

Definition 28 We define $c_2Eordinal_2EallOrds$ to be $\lambda A_27a : \iota. (ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ t))$.

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS \\ & \quad A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)})}) \end{aligned} \tag{13}$$

Definition 29 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota. \lambda V0r \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 30 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2E2ET)$.

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\forall V2x \in A_27a.(p (ap V0P V2x))) \wedge (p V1Q)) \Leftrightarrow (\forall V3x \in A_27a.((p (ap V0P V3x)) \wedge (p V1Q)))))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_27a.(p (ap V1Q V3x)))))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \wedge (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))) \quad (24)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow ((\text{ap } (c_2Ewellorder_2EelsOf (ty_2Eordinal_2Eordinal A_27a)) (c_2Eordinal_2EallOrds A_27a)) = (c_2Epred_set_2EUNIV (ty_2Eordinal_2Eordinal A_27a))) \quad (25)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)). (p (\text{ap } (\text{ap } (c_2Ewellorder_2Eorderiso (ty_2Esum_2Esum ty_2Enum_2Enum A_27a) (ty_2Eordinal_2Eordinal A_27a)) V0w) (\text{ap } (\text{ap } (c_2Ewellorder_2Ewobound (ty_2Eordinal_2Eordinal A_27a)) (\text{ap } (c_2Eordinal_2Eordinal_ABS A_27a) V0w)) (c_2Eordinal_2EallOrds A_27a)))))) \quad (26)$$

Assume the following.

$$\forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (p (\text{ap } (\text{ap } (c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EUNIV A_27a)))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. (((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (32)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{38}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{39}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{40}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{41}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow \forall A.27c. \\
& nonempty A.27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder A.27a). \\
& (\forall V1w2 \in (ty_2Ewellorder_2Ewellorder A.27b). (\forall V2w3 \in \\
& (ty_2Ewellorder_2Ewellorder A.27c). (((p (ap (ap (c_2Ewellorder_2Eorderiso \\
& A.27a A.27b) V0w1) V1w2)) \wedge (p (ap (ap (c_2Ewellorder_2Eorderiso \\
& A.27b A.27c) V1w2) V2w3))) \Rightarrow (p (ap (ap (c_2Ewellorder_2Eorderiso \\
& A.27a A.27c) V0w1) V2w3))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder\ A.27a).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A.27a\ A.27a)\ V0w)\ V0w)) \Leftrightarrow False)) \quad (44)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0f \in (A.27a^{A.27b}).(\forall V1wo \in (ty_2Ewellorder_2Ewellorder\ A.27b).((p\ (ap\ (ap\ (ap\ (c_2Epred_set_2EINJ\ A.27b\ A.27a)\ V0f)\ (ap\ (c_2Ewellorder_2EelsOf\ A.27b)\ V1wo))\ (c_2Epred_set_2EUNIV\ A.27a))) \Rightarrow (\exists V2wo.27 \in (ty_2Ewellorder_2Ewellorder\ A.27a). (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A.27b\ A.27a)\ V1wo)\ V2wo.27)))))) \quad (45)$$

Theorem 1

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\neg (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ (ty_2Eordinal_2Eordinal\ A.27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a))\ (c_2Epred_set_2EUNIV\ (ty_2Eordinal_2Eordinal\ A.27a))))\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a))))))$$