

thm_2Eordinal_2Ewellorder__ordinal__isomorphism (TMX5cJXpYqHV6xnimnb3g5WM41otbuAtWTc)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}}))$

Definition 4 We define `c_2Emarker_2EAbbrev` to be $\lambda V0x \in 2.V0x$.

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty } \text{ty_2Enum_2Enum} \tag{1}$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty_2Esum_2Esum } A0 A1) \tag{2}$$

Let `ty_2Ewellorder_2Ewellorder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Ewellorder_2Ewellorder } A0) \tag{3}$$

Let `ty_2Eordinal_2Eordinal` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty } (\text{ty_2Eordinal_2Eordinal } A0) \tag{4}$$

Let `c_2Eordinal_2Eordinal__REP__CLASS` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$A_{27a} \in ((2^{(\text{ty_2Ewellorder_2Ewellorder } (\text{ty_2Esum_2Esum } \text{ty_2Enum_2Enum } A_{27a}))} (\text{ty_2Eordinal_2Eordinal } A_{27a}))) \tag{5}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** $(the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 6 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal A_27a$.
Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A_27a \in ((2^{(ty_2Epair_2Eprod A_27a A_27a)})^{(ty_2Ewellorder_2Ewellorder A_27a)}) \quad (7)$$

Definition 7 We define c_2Ebool_2E2F to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E2F$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 11 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Definition 12 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (10)$$

Definition 13 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (11)$$

Definition 14 We define $c_2Eset_relation_2Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Definition 15 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 16 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$.

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (12)$$

Definition 17 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ 2)\ P)))$.

Definition 19 We define $c_2Eset_relation_2Erangle$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 20 We define $c_2Eset_relation_2Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$.

Definition 21 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)))$.

Definition 22 We define $c_2Epred_set_2EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Emin_2E_40\ 2)\ s\ t))$.

Definition 23 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 24 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 25 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 26 We define $c_2Eordinal_2Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a)$.

Definition 27 We define $c_2Eordinal_2EallOrds$ to be $\lambda A_27a : \iota.(ap\ (c_2Ewellorder_2Ewellorder_ABS\ A_27a)\ T1)$.

Let $c_2Eordinal_2Eordinal_ABS_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Eordinal_2Eordinal_ABS_CLASS \\ A_27a \in ((ty_2Eordinal_2Eordinal\ A_27a)^{(2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Eenum_2Eenum\ A_27a))})}) \end{aligned} \quad (13)$$

Definition 28 We define $c_2Eordinal_2Eordinal_ABS$ to be $\lambda A_27a : \iota.\lambda V0r \in (ty_2Ewellorder_2Ewellorder\ A_27a)$.

Definition 29 We define $c_2Equotient_2EQUOTIENT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27a})^{A_27b})$.

Definition 30 We define $c_Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (21)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\forall V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). ((\neg(\exists V1x \in A_27a. (p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A_27a. (\neg(p\ (ap\ V0P\ V2x)))))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p\ V0A) \Rightarrow (p\ V1B))) \Leftrightarrow ((p\ V0A) \wedge \neg(p\ V1B)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V1B) \wedge (p\ V2C) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \quad (30)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (31)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Equotient_2EQUOTIENT\ (ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ (ty_2Eordinal_2Eordinal\ A_27a))\ (c_2Ewellorder_2Eorderiso\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ (c_2Eordinal_2Eordinal_ABS\ A_27a))\ (c_2Eordinal_2Eordinal_REP\ A_27a)))) \quad (33)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a).(\forall V1y \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap \\
& \quad (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ (ty_2Eordinal_2Eordinal \\
& \quad A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a)))\ (ap\ (ap\ (c_2Epair_2E_2C \\
& \quad (ty_2Eordinal_2Eordinal\ A_27a)\ (ty_2Eordinal_2Eordinal\ A_27a))) \\
& \quad V0x)\ V1y))\ (ap\ (c_2Eset_relation_2Estrict\ (ty_2Eordinal_2Eordinal \\
& \quad A_27a))\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ (ty_2Eordinal_2Eordinal \\
& \quad A_27a))\ (c_2Eordinal_2EallOrds\ A_27a)))))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& \quad A_27a)\ V0x)\ V1y))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0o1 \in (ty_2Eordinal_2Eordinal \\
& \quad A_27a).(\forall V1o2 \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap \\
& \quad (ap\ (c_2Eordinal_2Eordlt\ A_27a)\ V0o1)\ V1o2)) \Leftrightarrow (\forall V2w1 \in (\\
& \quad ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum \\
& \quad A_27a)).(\forall V3w2 \in (ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum \\
& \quad ty_2Enum_2Enum\ A_27a)).((((ap\ (c_2Eordinal_2Eordinal_ABS \\
& \quad A_27a)\ V2w1) = V0o1) \wedge ((ap\ (c_2Eordinal_2Eordinal_ABS\ A_27a) \\
& \quad V3w2) = V1o2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ (ty_2Esum_2Esum \\
& \quad ty_2Enum_2Enum\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)) \\
& \quad V2w1)\ V3w2)))))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in \\
& \quad (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso \\
& \quad A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt \\
& \quad A_27a\ A_27b)\ V0w1)\ V1w2))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1w \in \\
& \quad (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V2y \in A_27a.((p \\
& \quad (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27a)\ (ap\ (ap\ (c_2Ewellorder_2Ewobound \\
& \quad A_27a)\ V0x)\ V1w))\ (ap\ (ap\ (c_2Ewellorder_2Ewobound\ A_27a)\ V2y) \\
& \quad V1w))) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ (ty_2Epair_2Eprod\ A_27a\ A_27a)) \\
& \quad (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27a)\ V0x)\ V2y))\ (ap\ (c_2Eset_relation_2Estrict \\
& \quad A_27a)\ (ap\ (c_2Ewellorder_2Ewellorder_REP\ A_27a)\ V1w)))))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b))) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (38) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \hspace{15em} (39) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (42) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \quad (43) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \quad (45) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \wedge (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((\neg(p\ V1q)) \vee (\neg(p\ V2r)))) \wedge (((p\ V1q) \vee \\ & \quad (\neg(p\ V0p))) \wedge ((p\ V2r) \vee (\neg(p\ V0p)))))))))) \quad (46) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee \neg(p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee \neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q) \vee ((p \ V2r) \vee \neg(p \ V0p))))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow \neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge (\neg(p \ V1q) \vee \neg(p \ V0p))))))
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p))) \tag{50}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow \neg(p \ V1q))) \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0a \in A_27a. (\forall V1b \in \\
& A_27a. (\forall V2w \in (\text{ty_2Ewellorder_2Ewellorder } A_27a). ((p \\
& (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } (\text{ty_2Epair_2Eprod } A_27a \ A_27a)) (\text{ap } (\text{ap } \\
& (\text{c_2Epair_2E_2C } A_27a \ A_27a) \ V0a) \ V1b)) (\text{ap } (\text{c_2Eset_relation_2Estrict} \\
& A_27a) (\text{ap } (\text{c_2Ewellorder_2Ewellorder_REP } A_27a) \ V2w)))) \Rightarrow (\\
& (\text{ap } (\text{ap } (\text{c_2Ewellorder_2Ewobound } A_27a) \ V0a) (\text{ap } (\text{ap } (\text{c_2Ewellorder_2Ewobound} \\
& A_27a) \ V1b) \ V2w)) = (\text{ap } (\text{ap } (\text{c_2Ewellorder_2Ewobound } A_27a) \ V0a) \\
& \ V2w))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1w \in \\
& (\text{ty_2Ewellorder_2Ewellorder } A_27a). ((\text{ap } (\text{c_2Ewellorder_2EelsOf} \\
& A_27a) (\text{ap } (\text{ap } (\text{c_2Ewellorder_2Ewobound } A_27a) \ V0x) \ V1w)) = (\text{ap } \\
& (\text{c_2Epred_set_2EGSPEC } A_27a \ A_27a) (\lambda V2y \in A_27a. (\text{ap } (\text{ap } (\\
& \text{c_2Epair_2E_2C } A_27a \ 2) \ V2y) (\text{ap } (\text{ap } (\text{c_2Ebool_2EIN } (\text{ty_2Epair_2Eprod} \\
& A_27a \ A_27a)) (\text{ap } (\text{ap } (\text{c_2Epair_2E_2C } A_27a \ A_27a) \ V2y) \ V0x)) (\text{ap } \\
& (\text{c_2Eset_relation_2Estrict } A_27a) (\text{ap } (\text{c_2Ewellorder_2Ewellorder_REP} \\
& A_27a) \ V1w))))))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a).(p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27a)\ V0w)\ V0w))) \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0w1)\ V1w2)) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27b\ A_27a)\ V1w2)\ V0w1)))))) \quad (55)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2w3 \in (ty_2Ewellorder_2Ewellorder\ A_27c).(((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0w1)\ V1w2)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27b\ A_27c)\ V1w2)\ V2w3))) \Rightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27c)\ V0w1)\ V2w3)))))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1w2 \in (ty_2Ewellorder_2Ewellorder\ A_27b).((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27b)\ V0w1)\ V1w2)) \vee ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0w1)\ V1w2)) \vee (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27b\ A_27a)\ V1w2)\ V0w1)))))) \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (p\ (ap\ (c_2Erelation_2EWF\ (ty_2Ewellorder_2Ewellorder\ A_27a))\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27a))) \quad (58)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.nonempty\ A_27c \Rightarrow \forall A_27d.nonempty\ A_27d \Rightarrow (\forall V0x0 \in (ty_2Ewellorder_2Ewellorder\ A_27a).(\forall V1y0 \in (ty_2Ewellorder_2Ewellorder\ A_27b).(\forall V2a0 \in (ty_2Ewellorder_2Ewellorder\ A_27c).(\forall V3b0 \in (ty_2Ewellorder_2Ewellorder\ A_27d).(((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27a\ A_27b)\ V0x0)\ V1y0)) \wedge (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso\ A_27c\ A_27d)\ V2a0)\ V3b0))) \Rightarrow ((p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27a\ A_27c)\ V0x0)\ V2a0)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ewellorder_2Eorderlt\ A_27b\ A_27d)\ V1y0)\ V3b0)))))) \quad (59)$$

Theorem 1

$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0w \in (ty_2Ewellorder_2Ewellorder$
 $(ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a)).(p\ (ap\ (ap\ (c_2Ewellorder_2Eorderiso$
 $(ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a)\ (ty_2Eordinal_2Eordinal$
 $A.27a))\ V0w)\ (ap\ (ap\ (c_2Ewellorder_2Ewobound\ (ty_2Eordinal_2Eordinal$
 $A.27a))\ (ap\ (c_2Eordinal_2Eordinal_ABS\ A.27a)\ V0w))\ (c_2Eordinal_2EallOrds$
 $A.27a))))))$