thm_2Epair_2ELET2__RATOR (TMKZNSxBdc11jY5qeEi128426VHi76Jyd9r)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27a$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{2}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27}$

Definition 5 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27b}$

Assume the following.

$$True \tag{4}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{6}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1v \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ V1v) = (ap\ (ap\ V0f\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V1v))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V1v))))) \tag{7}$$

Theorem 1

$$\forall A\_27a1.nonempty\ A\_27a1 \Rightarrow \forall A\_27a2.nonempty\ A\_27a2 \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0M \in (ty\_2Epair\_2Eprod\ A\_27a1\ A\_27a2).(\forall V1N \in (((A\_27c^{A\_27b})^{A\_27a2})^{A\_27a1}).(\forall V2b \in A\_27b.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Epair\_2Eprod\ A\_27a1\ A\_27a2)\ (A\_27c^{A\_27b}))\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a1\ A\_27a2\ (A\_27c^{A\_27b}))\ (\lambda V3x \in A\_27a1.(\lambda V4y \in A\_27a2.(ap\ (ap\ V1N\ V3x)\ V4y)))))\ V0M)\ V2b) = (ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ (ty\_2Epair\_2Eprod\ A\_27a1\ A\_27a2)\ A\_27c)\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a1\ A\_27a2\ A\_27c)\ (\lambda V5x \in A\_27a1.(\lambda V6y \in A\_27a2.(ap\ (ap\ (ap\ V1N\ V5x)\ V6y)\ V2b)))))\ V0M)))))$$