

thm_2Epair_2EPFORALL_THM
(TMaTSKgnLycva1xfNRyiqx1kz9XWvvnoven)

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Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2ESND\ A.27a\ A.27b \in (A.27b^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (2)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow c_2Epair_2EFST\ A.27a\ A.27b \in (A.27a^{(ty_2Epair_2Eprod\ A.27a\ A.27b)}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Definition 4 We define $c_2Epair_2EUNCURRY$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c^{A-27b})^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x)))$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (4)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\ & nonempty\ A.27c \Rightarrow (\forall V0f \in ((A.27c^{A-27b})^{A-27a}).((ap\ (c_2Epair_2EUNCURRY\ A.27a\ A.27b\ A.27c)\ V0f) = (\lambda V1x \in (ty_2Epair_2Eprod\ A.27a\ A.27b). \\ & (ap\ (ap\ V0f\ (ap\ (c_2Epair_2EFST\ A.27a\ A.27b)\ V1x))\ (ap\ (c_2Epair_2ESND\ A.27a\ A.27b)\ V1x)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1p \in (ty_2Epair_2Eprod \\
& A_27a\ A_27b).(p\ (ap\ (ap\ V0P\ (ap\ (c_2Epair_2EFST\ A_27a\ A_27b)\ V1p)) \\
& (ap\ (c_2Epair_2ESND\ A_27a\ A_27b)\ V1p)))) \Leftrightarrow (\forall V2p1 \in A_27a. \\
& \quad (\forall V3p2 \in A_27b.(p\ (ap\ (ap\ V0P\ V2p1)\ V3p2))))))
\end{aligned} \tag{6}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0P \in ((2^{A_27b})^{A_27a}).((\forall V1x \in A_27a.(\forall V2y \in \\
& A_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (p\ (ap\ (c_2Ebool_2E_21\ (ty_2Epair_2Eprod \\
& A_27a\ A_27b))\ (ap\ (c_2Epair_2EUNCURRY\ A_27a\ A_27b\ 2)\ (\lambda V3x \in \\
& \quad A_27a.(\lambda V4y \in A_27b.(ap\ (ap\ V0P\ V3x)\ V4y))))))
\end{aligned}$$