

# thm\_2Epair\_2EUNCURRY\_CONG (TMFRsQ4YdctPkEetUUcbxhp9biwiCLC4RQt)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2T` to be  $(\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

**Definition 3** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D } (2^{A-27a}))$

**Definition 4** We define `c_2Ebool_2E_2F` to be  $(\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V0t \in 2. V0t))$ .

**Definition 5** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (P \Rightarrow Q)$  of type  $\iota$ .

**Definition 6** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c\_2Emin\_2E\_3D\_3D\_3E } V0t) \text{ c\_2Ebool\_2E\_2F}$

**Definition 7** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 8** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c\_2Ebool\_2E\_21 } 2) (\lambda V2t \in 2. V2t))$

**Definition 9** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. P \text{ (ap } P \ x)) \text{ then } (the (\lambda x. x \in A \wedge P \ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P (\text{ap } (\text{c\_2Emin\_2E\_40}$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty } (\text{ty\_2Epair\_2Eprod } A0 \ A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A. 27a. \text{nonempty } A. 27a \Rightarrow \forall A. 27b. \text{nonempty } A. 27b \Rightarrow \text{c\_2Epair\_2EABS\_prod } A. 27a \ A. 27b \in ((\text{ty\_2Epair\_2Eprod } A. 27a \ A. 27b))^{((2^{A-27b})^{A-27a})} \tag{2}$$

**Definition 11** We define `c_2Epair_2E_2C` to be  $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0x \in A. 27a. \lambda V1y \in A. 27b. (\text{ap } (\text{c\_2Emin\_2E\_3D}$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (3)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (4)$$

**Definition 12** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ p\ V0t)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ( \\ (p\ V1B) \vee (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \vee (p\ V2C)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \vee (p\ V1B)) \Leftrightarrow ((p\ V1B) \vee \\ (p\ V0A)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ (p\ V1B)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (12)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x\_27)) \wedge ((p \ V1x\_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y\_27)))) \Rightarrow \\ ((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x\_27) \Rightarrow (p \ V3y\_27)) \quad (13)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}). (\forall V1v \in \\ A\_27a. ((\forall V2x \in A\_27a. ((V2x = V1v) \Rightarrow (p \ (ap \ V0f \ V2x)))) \Leftrightarrow (p \ ( \\ ap \ V0f \ V1v)))))) \quad (14)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ \forall V0x \in A\_27a. (\forall V1y \in A\_27b. (\forall V2a \in A\_27a. (\forall V3b \in \\ A\_27b. (((ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V0x) \ V1y) = (ap \ (ap \\ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \quad (15)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ \forall V0x \in (ty\_2Epair\_2Eprod \ A\_27a \ A\_27b). (\exists V1q \in A\_27a. \\ (\exists V2r \in A\_27b. (V0x = (ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \\ V1q) \ V2r)))))) \quad (16)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\ \text{nonempty } A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1x \in \\ A\_27a. (\forall V2y \in A\_27b. ((ap \ (ap \ (c\_2Epair\_2EUNCURRY \ A\_27a \\ A\_27b \ A\_27c) \ V0f) \ (ap \ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V1x) \ V2y))) = \\ (ap \ (ap \ V0f \ V1x) \ V2y)))))) \quad (17)$$

### Theorem 1

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \forall A\_27c. \\ \text{nonempty } A\_27c \Rightarrow (\forall V0f\_27 \in ((A\_27c^{A\_27b})^{A\_27a}). (\forall V1f \in \\ ((A\_27c^{A\_27b})^{A\_27a}). (\forall V2M\_27 \in (ty\_2Epair\_2Eprod \ A\_27a \\ A\_27b). (\forall V3M \in (ty\_2Epair\_2Eprod \ A\_27a \ A\_27b). (((V3M = \\ V2M\_27) \wedge (\forall V4x \in A\_27a. (\forall V5y \in A\_27b. ((V2M\_27 = (ap \\ (ap \ (c\_2Epair\_2E\_2C \ A\_27a \ A\_27b) \ V4x) \ V5y)) \Rightarrow ((ap \ (ap \ V1f \ V4x) \ V5y) = \\ (ap \ (ap \ V0f\_27 \ V4x) \ V5y)))))) \Rightarrow ((ap \ (ap \ (c\_2Epair\_2EUNCURRY \ A\_27a \\ A\_27b \ A\_27c) \ V1f) \ V3M) = (ap \ (ap \ (c\_2Epair\_2EUNCURRY \ A\_27a \ A\_27b \\ A\_27c) \ V0f\_27) \ V2M\_27)))))))))$$