

thm_2Epair_2Epair__case__cong (TMUjcJkLgkUaB169ximejPeWiajGDhfmCy)

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Definition 1 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}}))$

Definition 5 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 6 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p (ap P x))$ *)* of type $\iota \Rightarrow \iota$.

Definition 7 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P)))$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{1}$$

Let `c_2Epair_2EABS_prod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2EABS_prod A_{27a} A_{27b} \in ((ty_2Epair_2Eprod A_{27a} A_{27b})^{(2^{A_{27b}})^{A_{27a}}}) \tag{2}$$

Definition 8 We define `c_2Epair_2E_2C` to be $\lambda A_{27a} : \iota.\lambda A_{27b} : \iota.\lambda V0x \in A_{27a}.\lambda V1y \in A_{27b}.(ap (c_2Emin_2E_40 A_{27a} (ap V0x V1y)))$

Let `c_2Epair_2ESND` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_{27a}.nonempty A_{27a} \Rightarrow \forall A_{27b}.nonempty A_{27b} \Rightarrow c_2Epair_2ESND A_{27a} A_{27b} \in (A_{27b}^{(ty_2Epair_2Eprod A_{27a} A_{27b})}) \tag{3}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (4)$$

Definition 9 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_2Eprod\ A_27a\ A_27b)$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0x \in (ty_2Epair_2Eprod\ A_27a\ A_27b).(\exists V1q \in A_27a. \\ (\exists V2r \in A_27b.(V0x = (ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b) \\ V1q)\ V2r)))))) \end{aligned} \quad (5)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b.(\forall V1y \in A_27c.(\forall V2f \in \\ ((A_27a^{A_27c})^{A_27b}).((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b \\ A_27c)\ (ap\ (ap\ (c_2Epair_2E_2C\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap \\ (ap\ V2f\ V0x)\ V1y)))))) \end{aligned} \quad (6)$$

Theorem 1

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f_27 \in ((A_27a^{A_27c})^{A_27b}).(\forall V1M \in \\ (ty_2Epair_2Eprod\ A_27b\ A_27c).(\forall V2M_27 \in (ty_2Epair_2Eprod \\ A_27b\ A_27c).(\forall V3f \in ((A_27a^{A_27c})^{A_27b}).(((V1M = V2M_27) \wedge \\ (\forall V4x \in A_27b.(\forall V5y \in A_27c.((V2M_27 = (ap\ (ap\ (c_2Epair_2E_2C \\ A_27b\ A_27c)\ V4x)\ V5y)) \Rightarrow ((ap\ (ap\ V3f\ V4x)\ V5y) = (ap\ (ap\ V0f_27\ V4x) \\ V5y)))))) \Rightarrow ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b\ A_27c) \\ V1M)\ V3f) = (ap\ (ap\ (c_2Epair_2Epair_CASE\ A_27a\ A_27b\ A_27c)\ V2M_27) \\ V0f_27)))))) \end{aligned}$$