thm_2Epath_2ELTAKE__labels (TMRaYe8SSS6nFgCZTFLxfBujuaJG2vNiLCN)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 4 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 8 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{8}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{9}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{10}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{11}$$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{12}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{13}$$

Definition 12 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (14)$$

Definition 13 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 14 *We define* c_2Eone_2Eone *to be* $(ap\ (\mathsf{c\_2Emin\_2E\_40}\ ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 15 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 16 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 17 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 18 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 19 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (15)$$

Definition 20 *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (16)$$

Let $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum}) \quad (17)$$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (18)$$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (19)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (20)$$

Definition 21 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 22 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 24 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 26 *We define* c_2Ellist_2Ellength__rel *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\lambda V$

Definition 27 *We define* c_2Ellist_2ELFINITE *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap$ (c

Definition 28 *We define* c_2Ellist_2ELLENGTH *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (a$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \tag{21}$$

Definition 29 *We define* c_2Ellist_2EtoList *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{22}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{23}$$

Let $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath\ A0\ A1) \tag{24}$$

Let $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{25}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{27}$$

Definition 30 *We define* c_2Epath_2Efinite *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0sigma \in (ty\_2Epath\_2Epath\ A$

Definition 31 *We define* c_2Epath_2Elength *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a$

Definition 32 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{28}$$

Definition 33 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{29}$$

Definition 34 *We define* c_2Epath_2EPL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Let $c\_2Epath\_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Etail\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{30}$$

Let $c\_2Epath\_2Efirst\_\_label : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Efirst\_\_label\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{31}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{32}$$

Definition 35 *We define* c_2Epath_2Efirst *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Let $c\_2Epath\_2Etake : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Etake\ A\_27a\ A\_27b \in (((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)})^{ty\_2Enum\_2Enum}) \tag{33}$$

Let $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EtoPath\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod} \tag{34}$$

Definition 36 *We define* c_2Epath_2Epcons *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1r \in A\_27b.\lambda V2p$

Definition 37 *We define* c_2Epath_2Estopped__at *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Epath$

Let $c\_2Epath\_2Elabels : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epath\_2Elabels\ A\_27a\ A\_27b \in ((ty\_2Ellist\_2Ellist\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{35}$$

Definition 38 *We define* c_2Epred__set_2EEMPTY *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2EF})$.

Definition 39 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 40 *We define* c_2Epred__set_2EINSERT *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap\ (c\_$

Definition 41 *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

Assume the following.

$$True \tag{36}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\wedge True){\Leftrightarrow}(p\ V0t))\wedge(((False\wedge(p\ V0t)){\Leftrightarrow}False)\wedge((((p\ V0t)\wedge False){\Leftrightarrow}False)\wedge(((p\ V0t)\wedge(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True\vee(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t)\vee True){\Leftrightarrow}True)\wedge(((False\vee(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t)\vee False){\Leftrightarrow}(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t)){\Leftrightarrow}(p\ V0t))))))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t))\wedge((((p\ V0t){\Rightarrow}True){\Leftrightarrow}True)\wedge(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True)\wedge(((p\ V0t){\Rightarrow}False){\Leftrightarrow}(\neg(p\ V0t)))))))) \tag{41}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t)))\wedge(((\neg True){\Leftrightarrow}False)\wedge((\neg False){\Leftrightarrow}True))) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{48}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \wedge (\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x))))))) \tag{49}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{50}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{52}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V0h))))) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Ellist\_2Ellist\ A\_27a))) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V1t))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1n \in ty\_2Enum\_2Enum.(\forall V2ll \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2ll) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ V2ll))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V3hd \in A\_27a.(ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Elist\_2Elist\ A\_27a)\ (ty\_2Eoption\_2Eoption\ (ty\_2Elist\_2Elist\ A\_27a)))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ V1n)\ (ap\ (c\_2Eoption\_2ETHE\ (ty\_2Ellist\_2Ellist\ A\_27a))\ (ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ V2ll))))\ (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27a)))\ (\lambda V4tl \in (ty\_2Elist\_2Elist\ A\_27a).(ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3hd)\ V4tl)))))))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow ((\forall V0l \in (ty\_2Ellist\_2Ellist\ A\_27a).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27a)\ \mathsf{c\_2Enum\_2E0})\ V0l) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Elist\_2Elist\ A\_27b)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3h \in A\_27c.(\forall V4t \in (ty\_2Ellist\_2Ellist\ A\_27c).((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27c)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V2n))\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27c)\ V3h)\ V4t)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Elist\_2Elist\ A\_27c)\ (ty\_2Elist\_2Elist\ A\_27c))\ (ap\ (c\_2Elist\_2ECONS\ A\_27c)\ V3h))\ (ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27c)\ V2n)\ V4t)))))))) \tag{56}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2EtoList}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27a))\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2EtoList}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ (ty\_2Elist\_2Elist\ A\_27b)\ (ty\_2Elist\_2Elist\ A\_27b))\ (ap\ (c\_2Elist\_2ECONS\ A\_27b)\ V0h))\ (ap\ (\mathsf{c\_2Ellist\_2EtoList}\ A\_27b)\ V1t)))))) \tag{57}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{58}$$

Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0})) \wedge (\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n))))) \tag{59}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0opt \in (ty\_2Eoption\_2Eoption\ A\_27a).((V0opt = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \vee (\exists V1x \in A\_27a.(V0opt = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (c\_2Eoption\_2ETHE\ A\_27a)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)) = V0x)) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in (ty\_2Eoption\_2Eoption\ A\_27a).(\forall V2y \in A\_27b.(((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V2y)) \Leftrightarrow (\exists V3z \in A\_27a.((V1x = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V3z)) \wedge (V2y = (ap\ V0f\ V3z)))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27a^{A\_27b}).(\forall V1x \in (ty\_2Eoption\_2Eoption\ A\_27b).((((ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27b\ A\_27a)\ V0f)\ V1x) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow (V1x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b))) \wedge (((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (ap\ (c\_2Eoption\_2EOPTION\_\_MAP\ A\_27b\ A\_27a)\ V0f)\ V1x)) \Leftrightarrow (V1x = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27b)))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((\exists V1x \in A\_27a. (V0p = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V1x))) \vee (\exists V2x \in A\_27a.(\exists V3r \in A\_27b.(\exists V4q \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(V0p = (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V2x)\ V3r)\ V4q))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p)) = V2p)))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Efirst\_\_label\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p)) = V1r)))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Epath\_2EPL}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ ty\_2Enum\_2Enum)))) \wedge (\forall V1x \in A\_27a.(\forall V2r \in A\_27b.(\forall V3q \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (\mathsf{c\_2Epath\_2EPL}\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3q)) = (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2ESUC})\ (ap\ (\mathsf{c\_2Epath\_2EPL}\ A\_27a\ A\_27b)\ V3q)))))))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})\ (ap\ (\mathsf{c\_2Epath\_2EPL}\ A\_27a\ A\_27b)\ V0p)))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}((\forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (ap\ (c\_2Epath\_2Etake\ A\_27a\ A\_27b)\ \mathsf{c\_2Enum\_2E0})\ V0p) = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ V0p))))\wedge(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b). ((ap\ (ap\ (c\_2Epath\_2Etake\ A\_27a\ A\_27b)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V2p) = (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ V2p))\ (ap\ (c\_2Epath\_2Efirst\_\_label\ A\_27a\ A\_27b)\ V2p))\ (ap\ (ap\ (c\_2Epath\_2Etake\ A\_27a\ A\_27b)\ V1n)\ (ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ V2p))))))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}((\forall V0x \in A\_27a.((ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b)))\wedge(\forall V1x \in A\_27a.(\forall V2r \in A\_27b.(\forall V3p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3p)) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V2r)\ (ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ V3p))))))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (\mathsf{c\_2Epred\_\_set\_2EEMPTY}\ A\_27a))))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EINSERT}\ A\_27a)\ V1y)\ V2s)))\Leftrightarrow((V0x = V1y)\vee(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V2s))))))) \tag{75}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}(\forall V0y \in A\_27b.(\forall V1s \in (2^{A\_27a}).(\forall V2f \in (A\_27b^{A\_27a}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ V0y)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s)))\Leftrightarrow(\exists V3x \in A\_27a.((V0y = (ap\ V2f\ V3x))\wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V3x)\ V1s)))))))) \tag{76}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\Leftrightarrow(V0m = V1n)))) \tag{77}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{78}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg(p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee ((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\wedge(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((\neg(p\ V1q))\vee(\neg(p\ V2r))))\wedge(((p\ V1q)\vee (\neg(p\ V0p)))\wedge((p\ V2r)\vee(\neg(p\ V0p)))))))))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\vee(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(\neg(p\ V1q)))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge ((p\ V1q)\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow((p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee(p\ V1q))\wedge(((p\ V0p)\vee(\neg(p\ V2r)))\wedge((\neg(p\ V1q))\vee((p\ V2r)\vee(\neg(p\ V0p))))))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\vee (p\ V1q))\wedge((\neg(p\ V1q))\vee(\neg(p\ V0p))))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (p\ V0p)))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \Rightarrow (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V0p))))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p) \vee (p\ V1q))) \Rightarrow (\neg(p\ V1q))))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))) \Rightarrow (p\ V0p))) \tag{92}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(\forall V2l \in (ty\_2Elist\_2Elist\ A\_27b).(((ap\ (ap\ (c\_2Ellist\_2ELTAKE\ A\_27b)\ V0n)\ (ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ V1p)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27b))\ V2l)) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ ty\_2Enum\_2Enum)\ V0n)\ (ap\ (\mathsf{c\_2Epath\_2EPL}\ A\_27a\ A\_27b)\ V1p))) \wedge ((ap\ (\mathsf{c\_2Ellist\_2EtoList}\ A\_27b)\ (ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Epath\_2Etake\ A\_27a\ A\_27b)\ V0n)\ V1p))) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Elist\_2Elist\ A\_27b))\ V2l)))))))$$