thm_2Epath_2Ebuild__pcomp__trace (TMJKPLB-hvxMGWeie35htPABLqDy434cYw74)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.$c_2Ebool_2ET$)$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 6 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 9 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{7}$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{8}$$

Let $c\_2Ellist\_2Ellist\_\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{9}$$

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in ((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \tag{10}$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{11}$$

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{12}$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum\ A\_27a\ A\_27b \in ((ty\_2Esum\_2Esum\ A\_27a\ A\_27b)^{(((2^{A\_27b})^{A\_27a})^{2})}) \tag{13}$$

Definition 13 *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS\ A\_27a \in ((ty\_2Eoption\_2Eoption\ A\_27a)^{(ty\_2Esum\_2Esum\ A\_27a\ ty\_2Eone\_2Eone)}) \quad (14)$$

Definition 14 *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_\_$

Definition 15 *We define* c_2Eone_2Eone *to be* $(ap\ ($c_2Emin_2E_40 $ty\_2Eone\_2Eone)\ (\lambda V0x \in ty\_2Eone\_2$

Definition 16 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 18 *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap\ (c\_2Esum\_2EABS$

Definition 19 *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Eoption\_2Eoption\_\_ABS\ A\_27a)\ ($

Definition 20 *We define* c_2Ellist_2ELHD *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (c\_2$

Let $c\_2Eoption\_2Eoption\_\_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b \in (((A\_27b^{(A\_27b^{A\_27a})})^{A\_27b})^{(ty\_2Eoption\_2Eoption\ A\_27a)}) \quad (15)$$

Definition 21 *We define* c_2Ellist_2ELTL *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Definition 22 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (16)$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (17)$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (18)$$

Definition 23 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath\ A0\ A1) \quad (19)$$

Let $c\_2Epath\_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Etail\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (20)$$

Let $c\_2Epath\_2Efirst\_\_label : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Efirst\_\_label\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (21)$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (22)$$

Definition 24 *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A$

Definition 25 *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty$

Definition 26 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (23)$$

Definition 27 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epath\_2EtoPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EtoPath\ A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod} \quad (24)$$

Definition 28 *We define* c_2Epath_2Estopped__at *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Epath$

Definition 29 *We define* c_2Epath_2Eis__stopped *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A$

Let $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod\ A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \quad (25)$$

Definition 30 *We define* c_2Epath_2Efirst *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Definition 31 *We define* c_2Epath_2Epcons *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1r \in A\_27b.\lambda V2p$

Definition 32 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair\_$

Definition 33 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 34 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1g$

Let $c\_2Eoption\_2EOPTION\_\_BIND : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_BIND\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{A\_27b})})^{(ty\_2Eoption\_2Eoption\ A\_27b} \tag{26}$$

Let $c\_2Earithmetic\_2EFUNPOW : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Earithmetic\_2EFUNPOW\ A\_27a \in (((A\_27a^{A\_27a})^{ty\_2Enum\_2Enum})^{(A\_27a^{A\_27a})}) \tag{27}$$

Let $c\_2Eoption\_2EOPTION\_\_MAP : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Eoption\_2EOPTION\_\_MAP\ A\_27a\ A\_27b \in (((ty\_2Eoption\_2Eoption\ A\_27b)^{(ty\_2Eoption\_2Eoption\ A\_27a)})^{(A\_27b^{A\_27a})}) \tag{28}$$

Definition 35 *We define* c_2Ellist_2ELUNFOLD *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eoption\_2Eoptio$

Let $c\_2Epath\_2Elabels : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epath\_2Elabels\ A\_27a\ A\_27b \in ((ty\_2Ellist\_2Ellist\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{29}$$

Definition 36 *We define* c_2Epath_2Eunfold *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0proj \in (A\_27a^{A\_27c})$

Definition 37 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{30}$$

Definition 38 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 39 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 40 *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

Definition 41 *We define* c_2Epath_2Eokpath__f *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R\in(((2^{A\_27a})^{A\_27b})^{A\_27a}).$

Definition 42 *We define* c_2Epred__set_2ESUBSET *to be* $\lambda A\_27a:\iota.\lambda V0s\in(2^{A\_27a}).\lambda V1t\in(2^{A\_27a}).(ap\ ($

Definition 43 *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a:\iota.\lambda V0P\in(2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

Definition 44 *We define* c_2EfixedPoint_2Egfp *to be* $\lambda A_27a:\iota.\lambda V0f\in((2^{A_27a})^{(2^{A_27a})}).(ap\ ($c_2Epred__se

Definition 45 *We define* c_2Epath_2Eokpath *to be* $\lambda A\_27a:\iota.\lambda A\_27b:\iota.\lambda V0R\in(((2^{A\_27a})^{A\_27b})^{A\_27a}).(ap$

Let $c\_2Epath\_2Eparallel\_\_comp:\iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow\forall A\_27c.\ nonempty\ A\_27c\Rightarrow\forall A\_27d.nonempty\ A\_27d\Rightarrow\forall A\_27e.nonempty\ A\_27e\Rightarrow c\_2Epath\_2Eparallel\_\_comp\ A\_27a\ A\_27b\ A\_27c\ A\_27d\ A\_27e\in (((((2^{(ty\_2Epair\_2Eprod\ A\_27c\ A\_27e)})^{A\_27b})^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27d)})^{(((2^{A\_27e})^{A\_27b})^{A\_27d})})^{(((2^{A\_27c})^{A\_27b})^{A\_2} \tag{31}$$

Assume the following.

$$True \tag{32}$$

Assume the following.

$$(\forall V0t1\in 2.(\forall V1t2\in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \tag{33}$$

Assume the following.

$$(\forall V0t\in 2.(False\Rightarrow(p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0t\in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{35}$$

Assume the following.

$$(\forall V0t\in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow(p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge(((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{36}$$

Assume the following.

$$(\forall V0t\in 2.(((True\vee(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\vee True)\Leftrightarrow True)\wedge(((False\vee(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\vee False)\Leftrightarrow(p\ V0t))\wedge(((p\ V0t)\vee(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{37}$$

Assume the following.

$$(\forall V0t\in 2.(((True\Rightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Rightarrow True)\Leftrightarrow True)\wedge(((False\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge((((p\ V0t)\Rightarrow(p\ V0t))\Leftrightarrow True)\wedge(((p\ V0t)\Rightarrow False)\Leftrightarrow(\neg(p\ V0t)))))))) \tag{38}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t)))\wedge(((\neg True)\Leftrightarrow False)\wedge ((\neg False)\Leftrightarrow True))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(V0x = V0x)) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((V0x = V0x)\Leftrightarrow True)) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y)\Leftrightarrow(V1y = V0x)))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1)\wedge((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V0P\ V2x))\wedge(p\ (ap\ V1Q\ V2x))))\Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V0P\ V3x)))\wedge(\forall V4x \in A\_27a.(p\ (ap\ V1Q\ V4x))))))) \tag{45}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\forall V2x \in A\_27a.(p\ (ap\ V0P\ V2x)))\wedge(p\ V1Q))\Leftrightarrow(\forall V3x \in A\_27a.((p\ (ap\ V0P\ V3x))\wedge(p\ V1Q)))))) \tag{46}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P)\wedge(\forall V2x \in A\_27a.(p\ (ap\ V1Q\ V2x))))\Leftrightarrow(\forall V3x \in A\_27a.((p\ V0P)\wedge(p\ (ap\ V1Q\ V3x))))))) \tag{47}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x)))))) \tag{48}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{49}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{51}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p\ (ap\ V0P\ V2x)))) \Leftrightarrow (p\ (ap\ V0P\ V1a))))) \tag{52}$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)) \Leftrightarrow True)) \tag{53}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V0h))))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27a)\ (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Ellist\_2Ellist\ A\_27a))) \wedge (\forall V0h \in A\_27b.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27b).((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V0h)\ V1t)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V1t))))) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h \in A\_27a.(\forall V1t \in (ty\_2Ellist\_2Ellist\ A\_27a).((\neg((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a))) \wedge (\neg((\mathsf{c\_2Ellist\_2ELNIL}\ A\_27a) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h)\ V1t)))))) \quad (56)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0h1 \in A\_27a.(\forall V1t1 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\forall V2h2 \in A\_27a.(\forall V3t2 \in (ty\_2Ellist\_2Ellist\ A\_27a).(((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V0h1)\ V1t1) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27a)\ V2h2)\ V3t2)) \Leftrightarrow ((V0h1 = V2h2) \wedge (V1t1 = V3t2))))))) \quad (57)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0R \in ((2^{(ty\_2Ellist\_2Ellist\ A\_27b)})^{A\_27a}).(\forall V1f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))^{A\_27a}).(\forall V2s \in A\_27a.(\forall V3ll \in (ty\_2Ellist\_2Ellist\ A\_27b).(((p\ (ap\ (ap\ V0R\ V2s)\ V3ll)) \wedge (\forall V4s \in A\_27a.(\forall V5ll \in (ty\_2Ellist\_2Ellist\ A\_27b).((p\ (ap\ (ap\ V0R\ V4s)\ V5ll)) \Rightarrow ((((ap\ V1f\ V4s) = (\mathsf{c\_2Eoption\_2ENONE}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) \wedge (V5ll = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b))) \vee (\exists V6s\_27 \in A\_27a.(\exists V7x \in A\_27b.(\exists V8ll\_27 \in (ty\_2Ellist\_2Ellist\ A\_27b).(((ap\ V1f\ V4s) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V6s\_27)\ V7x))) \wedge (((ap\ (\mathsf{c\_2Ellist\_2ELHD}\ A\_27b)\ V5ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27b)\ V7x)) \wedge (((ap\ (\mathsf{c\_2Ellist\_2ELTL}\ A\_27b)\ V5ll) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Ellist\_2Ellist\ A\_27b))\ V8ll\_27)) \wedge (p\ (ap\ (ap\ V0R\ V6s\_27)\ V8ll\_27))))))))))))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27b\ A\_27a)\ V1f)\ V2s) = V3ll)))))) \quad (58)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0v \in A\_27b.(\forall V1f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2x \in A\_27a.(\forall V3v \in A\_27b.(\forall V4f \in (A\_27b^{A\_27a}).((ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2x))\ V3v)\ V4f) = (ap\ V4f\ V2x)))))) \quad (59)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1y)) \Leftrightarrow (V0x = V1y)))) \quad (60)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\neg((\mathsf{c\_2Eoption\_2ENONE}\ A\_27a) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V0x)))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow (\neg(p\ V0P))) \wedge ((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) \Leftrightarrow (p\ V0P)) \wedge ((((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x))\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2y)) \Leftrightarrow ((p\ V0P) \wedge (V1x = V2y))) \wedge (((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Eoption\_2Eoption\ A\_27a))\ V0P)\ (\mathsf{c\_2Eoption\_2ENONE}\ A\_27a))\ (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V1x)) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ A\_27a)\ V2y)) \Leftrightarrow ((\neg(p\ V0P)) \wedge (V1x = V2y))))))))) \tag{62}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.(\forall V2a \in A\_27a.(\forall V3b \in A\_27b.(((ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\exists V1q \in A\_27a.(\exists V2r \in A\_27b.(V0x = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1q)\ V2r))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in ((A\_27c^{A\_27b})^{A\_27a}).(\forall V1x \in A\_27a.(\forall V2y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Epair\_2EUNCURRY}\ A\_27a\ A\_27b\ A\_27c)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V1x)\ V2y)) = (ap\ (ap\ V0f\ V1x)\ V2y))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b.(\forall V1y \in A\_27c.(\forall V2f \in ((A\_27a^{A\_27c})^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ A\_27a\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((\exists V1x \in A\_27a. (V0p = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V1x))) \vee (\exists V2x \in A\_27a.(\exists V3r \in A\_27b.(\exists V4q \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(V0p = (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V2x)\ V3r)\ V4q))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)) = V0x)) \wedge (\forall V1x \in A\_27a.(\forall V2r \in A\_27b. (\forall V3p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3p)) = V1x))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p)) = V2p)))) \tag{69}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Efirst\_\_label\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p)) = V1r)))) \tag{70}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0x \in A\_27a.((ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)) = (\mathsf{c\_2Ellist\_2ELNIL}\ A\_27b))) \wedge (\forall V1x \in A\_27a.(\forall V2r \in A\_27b.(\forall V3p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1x)\ V2r)\ V3p)) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELCONS}\ A\_27b)\ V2r)\ (ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ V3p))))))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow ((\forall V0x \in A\_27a.\ ((p\ (ap\ (\mathsf{c\_2Epath\_2Eis\_\_stopped}\ A\_27a\ A\_27b)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x))) \Leftrightarrow True)) \wedge (\forall V1x \in A\_27c.(\forall V2r \in A\_27d.(\forall V3p \in (ty\_2Epath\_2Epath\ A\_27c\ A\_27d).((p\ (ap\ (\mathsf{c\_2Epath\_2Eis\_\_stopped}\ A\_27c\ A\_27d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27c\ A\_27d)\ V1x)\ V2r)\ V3p))) \Leftrightarrow False)))))) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\ \forall V0R \in (((2^{A\_27a})^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\ p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27a\ A\_27b)\ V0R)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V1x)))) \wedge (\forall V2x \in A\_27a.(\forall V3r \in A\_27b.\ (\forall V4p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27a\ A\_27b)\ V0R)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V2x)\ V3r)\ V4p))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0R\ V2x)\ V3r)\ (ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ V4p))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27a\ A\_27b)\ V0R)\ V4p))))))))) \tag{73}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0proj \in (A\_27b^{A\_27a}).(\forall V1f \in (\ (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27a}).\ (\forall V2s \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eunfold}\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f)\ V2s) = (ap\ (ap\ (ap\ (c\_2Eoption\_2Eoption\_\_CASE\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ (ty\_2Epath\_2Epath\ A\_27b\ A\_27c))\ (ap\ V1f\ V2s))\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27b\ A\_27c)\ (ap\ V0proj\ V2s)))\ (\lambda V3v \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c).(ap\ (ap\ (\mathsf{c\_2Epair\_2Epair\_\_CASE}\ (ty\_2Epath\_2Epath\ A\_27b\ A\_27c)\ A\_27a\ A\_27c)\ V3v)\ (\lambda V4s\_27 \in A\_27a.(\lambda V5l \in A\_27c.(ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27b\ A\_27c)\ (ap\ V0proj\ V2s))\ V5l)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eunfold}\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f)\ V4s\_27))))))))))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0proj \in (A\_27b^{A\_27a}).(\forall V1f \in (\ (ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27a}).\ (\forall V2s \in A\_27a.((ap\ (c\_2Epath\_2Elabels\ A\_27b\ A\_27c)\ (ap\ (\ ap\ (ap\ (\mathsf{c\_2Epath\_2Eunfold}\ A\_27b\ A\_27c\ A\_27a)\ V0proj)\ V1f)\ V2s)) = (ap\ (ap\ (\mathsf{c\_2Ellist\_2ELUNFOLD}\ A\_27c\ A\_27a)\ V1f)\ V2s))))) \tag{75}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1m \in (((2^{A\_27b})^{A\_27c})^{A\_27b}). (\forall V2proj \in (A\_27b^{A\_27a}).(\forall V3f \in ((ty\_2Eoption\_2Eoption\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))^{A\_27a}).(\forall V4s \in A\_27a.(((p\ (ap\ V0P\ V4s)) \wedge ((\forall V5s \in A\_27a.(\forall V6s\_27 \in A\_27a. (\forall V7l \in A\_27c.(((p\ (ap\ V0P\ V5s)) \wedge ((ap\ V3f\ V5s) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V6s\_27)\ V7l)))) \Rightarrow (p\ (ap\ V0P\ V6s\_27)))))) \wedge (\forall V8s \in A\_27a. (\forall V9s\_27 \in A\_27a.(\forall V10l \in A\_27c.(((p\ (ap\ V0P\ V8s)) \wedge ((ap\ V3f\ V8s) = (ap\ (\mathsf{c\_2Eoption\_2ESOME}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ V9s\_27)\ V10l)))) \Rightarrow (p\ (ap\ (ap\ (ap\ V1m\ (ap\ V2proj\ V8s))\ V10l)\ (ap\ V2proj\ V9s\_27))))))))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27b\ A\_27c)\ V1m)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eunfold}\ A\_27b\ A\_27c\ A\_27a)\ V2proj)\ V3f)\ V4s))))))))) \tag{76}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow \forall A\_27e.nonempty\ A\_27e \Rightarrow (\forall V0m1 \in (((2^{A\_27c})^{A\_27b})^{A\_27a}).(\forall V1m2 \in (((2^{A\_27e})^{A\_27b})^{A\_27d}).(\forall V2s1 \in A\_27a.(\forall V3s2 \in A\_27d.(\forall V4l \in A\_27b.(\forall V5s1\_27 \in A\_27c.(\forall V6s2\_27 \in A\_27e.((p\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Epath\_2Eparallel\_\_comp\ A\_27a\ A\_27b\ A\_27c\ A\_27d\ A\_27e)\ V0m1)\ V1m2)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27d)\ V2s1)\ V3s2))\ V4l)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27c\ A\_27e)\ V5s1\_27)\ V6s2\_27))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ V0m1\ V2s1)\ V4l)\ V5s1\_27)) \wedge (p\ (ap\ (ap\ (ap\ V1m2\ V3s2)\ V4l)\ V6s2\_27))))))))))) \tag{77}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{78}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{79}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{80}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))) \tag{81}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)){\Rightarrow}False){\Rightarrow}(((p\ V0A){\Rightarrow}False){\Rightarrow}False))) \tag{82}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}((p\ V1q){\Leftrightarrow}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}((p\ V1q){\vee}(p\ V2r))){\wedge}(((p\ V0p){\vee}((\neg(p\ V2r)){\vee}(\neg(p\ V1q)))){\wedge}(((p\ V1q){\vee}((\neg(p\ V2r)){\vee}(\neg(p\ V0p)))){\wedge}((p\ V2r){\vee}((\neg(p\ V1q)){\vee}(\neg(p\ V0p)))))))))) \tag{83}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}((p\ V1q){\wedge}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}((\neg(p\ V1q)){\vee}(\neg(p\ V2r)))){\wedge}(((p\ V1q){\vee}(\neg(p\ V0p))){\wedge}((p\ V2r){\vee}(\neg(p\ V0p))))))))) \tag{84}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}((p\ V1q){\vee}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}(\neg(p\ V1q))){\wedge}(((p\ V0p){\vee}(\neg(p\ V2r))){\wedge}((p\ V1q){\vee}((p\ V2r){\vee}(\neg(p\ V0p))))))))) \tag{85}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p){\Leftrightarrow}((p\ V1q){\Rightarrow}(p\ V2r))){\Leftrightarrow}(((p\ V0p){\vee}(p\ V1q)){\wedge}(((p\ V0p){\vee}(\neg(p\ V2r))){\wedge}((\neg(p\ V1q)){\vee}((p\ V2r){\vee}(\neg(p\ V0p))))))))) \tag{86}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p){\Leftrightarrow}(\neg(p\ V1q))){\Leftrightarrow}(((p\ V0p){\vee}(p\ V1q)){\wedge}((\neg(p\ V1q)){\vee}(\neg(p\ V0p))))))) \tag{87}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\Rightarrow}(p\ V1q))){\Rightarrow}(p\ V0p)))) \tag{88}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\Rightarrow}(p\ V1q))){\Rightarrow}(\neg(p\ V1q))))) \tag{89}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\vee}(p\ V1q))){\Rightarrow}(\neg(p\ V0p))))) \tag{90}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p){\vee}(p\ V1q))){\Rightarrow}(\neg(p\ V1q))))) \tag{91}$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p\ V0p))){\Rightarrow}(p\ V0p))) \tag{92}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\ nonempty\ A\_27c \Rightarrow (\forall V0m1 \in (((2^{A\_27a})^{A\_27b})^{A\_27a}).(\forall V1p1 \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(\forall V2m2 \in (((2^{A\_27c})^{A\_27b})^{A\_27c}).(\forall V3p2 \in (ty\_2Epath\_2Epath\ A\_27c\ A\_27b).(((p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27a\ A\_27b)\ V0m1)\ V1p1)) \wedge ((p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ A\_27c\ A\_27b)\ V2m2)\ V3p2)) \wedge ((ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ V1p1) = (ap\ (c\_2Epath\_2Elabels\ A\_27c\ A\_27b)\ V3p2)))) \Rightarrow (\exists V4p \in (ty\_2Epath\_2Epath\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eokpath}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ A\_27b)\ (ap\ (ap\ (c\_2Epath\_2Eparallel\_\_comp\ A\_27a\ A\_27b\ A\_27a\ A\_27c\ A\_27c)\ V0m1)\ V2m2))\ V4p)) \wedge (((ap\ (c\_2Epath\_2Elabels\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ A\_27b)\ V4p) = (ap\ (c\_2Epath\_2Elabels\ A\_27a\ A\_27b)\ V1p1)) \wedge ((ap\ (\mathsf{c\_2Epath\_2Efirst}\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27c)\ A\_27b)\ V4p) = (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27c)\ (ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27a\ A\_27b)\ V1p1))\ (ap\ (\mathsf{c\_2Epath\_2Efirst}\ A\_27c\ A\_27b)\ V3p2)))))))))))$$