

thm_2Epath_2Eel_def_compute
(TMWiRjHB4ixCKy9mEogz7yi4XZpy22Cd6FE)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2E2 to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2E0\ (c_2Enum_2EREP_num\ (c_2Enum_2ESUC_REP\ m))))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 6 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT2))$

Definition 7 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 8 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1))$

Definition 9 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Let $ty_2Epath_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epath_2Epath A0 A1) \quad (8)$$

Let $c_2Epath_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2Etail A_27a A_27b \in ((ty_2Epath_2Epath A_27a A_27b)^{(ty_2Epath_2Epath A_27a A_27b)}) \quad (9)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (10)$$

Let $ty_2Ellist_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ellist_2Ellist A0) \quad (11)$$

Let $c_2Epath_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epath_2EfromPath A_27a A_27b \in ((ty_2Epair_2Eprod A_27a (ty_2Ellist_2Ellist (ty_2Epair_2Eprod A_27b A_27a)))^{(ty_2Epath_2Epath A_27a A_27b)}) \quad (12)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \quad (13)$$

Definition 11 We define $c_2Epath_2Efirst$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (ty_2Epath_2Epath A_27a A_27b)$

Let $c_2Epath_2Eel : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epath_2Eel \\ A_27a\ A_27b \in ((A_27a^{(ty_2Epath_2Epath\ A_27a\ A_27b)})^{ty_2Enum_2Enum}) \end{aligned} \quad (14)$$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0f \in ((A_27a^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}). \\ (\forall V1g \in (A_27a^{ty_2Enum_2Enum}).(\forall V2n \in ty_2Enum_2Enum. \\ ((ap\ V1g\ (ap\ c_2Enum_2ESUC\ V2n)) = (ap\ (ap\ V0f\ V2n)\ (ap\ c_2Enum_2ESUC \\ V2n)))) \Leftrightarrow ((\forall V3n \in ty_2Enum_2Enum.((ap\ V1g\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ V3n))) = (ap\ (ap\ V0f\ (ap\ (ap\ c_2Earithmetic_2E_2D \\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V3n)))) \\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ V3n)))))) \wedge \\ (\forall V4n \in ty_2Enum_2Enum.((ap\ V1g\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT2\ V4n))) = (ap\ (ap\ V0f\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT1\ V4n)))\ (ap\ c_2Earithmetic_2ENUMERAL \\ (ap\ c_2Earithmetic_2EBIT2\ V4n)))))))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ (\forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Eel \\ A_27a\ A_27b)\ c_2Enum_2E0)\ V0p) = (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b) \\ V0p))) \wedge (\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in (ty_2Epath_2Epath \\ A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Eel\ A_27a\ A_27b)\ (ap\ c_2Enum_2ESUC \\ V1n))\ V2p) = (ap\ (ap\ (c_2Epath_2Eel\ A_27a\ A_27b)\ V1n)\ (ap\ (c_2Epath_2Etail \\ A_27a\ A_27b)\ V2p)))))) \end{aligned} \quad (17)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & (\forall V0p \in (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Eel \\ & A_27a\ A_27b)\ c_2Enum_2E0)\ V0p) = (ap\ (c_2Epath_2Efirst\ A_27a\ A_27b) \\ & V0p))) \wedge ((\forall V1n \in ty_2Enum_2Enum.(\forall V2p \in (ty_2Epath_2Epath \\ & A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Eel\ A_27a\ A_27b)\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ V1n)))\ V2p) = (ap\ (ap\ (c_2Epath_2Eel \\ & A_27a\ A_27b)\ (ap\ (ap\ c_2Earithmetic_2E_2D\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ V1n)))\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))\ (ap\ (c_2Epath_2Etail \\ & A_27a\ A_27b)\ V2p)))))) \wedge (\forall V3n \in ty_2Enum_2Enum.(\forall V4p \in \\ & (ty_2Epath_2Epath\ A_27a\ A_27b).((ap\ (ap\ (c_2Epath_2Eel\ A_27a \\ & A_27b)\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\ & V3n)))\ V4p) = (ap\ (ap\ (c_2Epath_2Eel\ A_27a\ A_27b)\ (ap\ c_2Earithmetic_2ENUMERAL \\ & (ap\ c_2Earithmetic_2EBIT1\ V3n)))\ (ap\ (c_2Epath_2Etail\ A_27a\ A_27b) \\ & V4p))))))))) \end{aligned}$$