# thm_2Epath_2Efilter__every (TMQBESdJE-BkjfVyLubDGJZEd9DC56vz5Zn9)

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**Definition 1** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
*of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
*of type* $\iota \Rightarrow \iota$.

**Definition 3** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 $A$

**Definition 4** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 5** *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 6** *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_27a}$

**Definition 7** *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ ($c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A\_2$

**Definition 8** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

**Definition 9** *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t)).$

**Definition 10** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
*of type* $\iota$.

**Definition 11** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E $V0t)\ $c_2Ebool_2E

**Definition 12** *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epath\_2Epath$$
$$A0\ A1)$$

$$(1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Epath\_2Eel : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2Eel$$
$$A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)})^{ty\_2Enum\_2Enum}) \qquad (3)$$

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**Definition 13** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap \ (\textsf{c\_2Ebool\_2E\_21} \ 2) \ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (6)$$

**Definition 14** *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap \ c\_2Enum\_2EABS\_\_num$

**Definition 15** *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty \ ty\_2Eone\_2Eone \qquad (7)$$

**Definition 16** *We define* c_2Eone_2Eone *to be* $(ap \ (\textsf{c\_2Emin\_2E\_40} \ ty\_2Eone\_2Eone) \ (\lambda V0x \in ty\_2Eone\_2$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow \forall A1.nonempty \ A1 \Rightarrow nonempty \ (ty\_2Esum\_2Esum \\ A0 \ A1) \qquad (8)$$

Let $c\_2Esum\_2EABS\_\_sum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow c\_2Esum\_2EABS\_\_sum \\ A\_27a \ A\_27b \in ((ty\_2Esum\_2Esum \ A\_27a \ A\_27b)^{(((2^{A\_27b})^{A\_27a})^2)}) \qquad (9)$$

**Definition 17** *We define* c_2Esum_2EINR *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap \ (c\_2Esum\_2EABS$

Let $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty \ A0 \Rightarrow nonempty \ (ty\_2Eoption\_2Eoption \ A0) \qquad (10)$$

Let $c\_2Eoption\_2Eoption\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow c\_2Eoption\_2Eoption\_\_ABS \ A\_27a \in \\ ((ty\_2Eoption\_2Eoption \ A\_27a)^{(ty\_2Esum\_2Esum \ A\_27a \ ty\_2Eone\_2Eone)}) \qquad (11)$$

**Definition 18** *We define* c_2Eoption_2ENONE *to be* $\lambda A\_27a : \iota.(ap \ (c\_2Eoption\_2Eoption\_\_ABS \ A\_27a) \ (a$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (12)$$

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**Definition 19** *We define* c_2Enum_2E0 *to be* (ap c_2Enum_2EABS__num c_2Enum_2EZERO__REP).

**Definition 20** *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

**Definition 21** *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 22** *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1) \tag{14}$$

Let $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ellist\_2Ellist\ A0) \tag{15}$$

Let $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epath\_2EfromPath$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod$$
$$A\_27b\ A\_27a)))^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}) \tag{16}$$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND$$
$$A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{17}$$

Let $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{18}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Let $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in$$
$$(((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{20}$$

**Definition 23** *We define* c_2Esum_2EINL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap\ (c\_2Esum\_2EABS$

**Definition 24** *We define* c_2Eoption_2ESOME *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Eoption\_2Eoption\_$

**Definition 25** *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Ellist\_2Ellist\_\_abs : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_\_abs\ A\_27a \in$$
$$((ty\_2Ellist\_2Ellist\ A\_27a)^{((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})}) \quad (21)$$

**Definition 26** *We define* c_2Ellist_2ELCONS *to be* $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist\ A.$

**Definition 27** *We define* c_2Ellist_2ELNIL *to be* $\lambda A\_27a : \iota.(ap\ (c\_2Ellist\_2Ellist\_\_abs\ A\_27a)\ (\lambda V0n \in ty\_$

**Definition 28** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

**Definition 29** *We define* c_2Ellist_2Ellength__rel *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(\lambda V$

**Definition 30** *We define* c_2Ellist_2ELFINITE *to be* $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ ($c

**Definition 31** *We define* c_2Ellist_2ELLENGTH *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (a$

Let $c\_2Eoption\_2ETHE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eoption\_2ETHE\ A\_27a \in (A\_27a^{(ty\_2Eoption\_2Eoption\ A\_27a)})$$
$$(22)$$

Let $c\_2Ellist\_2ELTAKE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2ELTAKE\ A\_27a \in (((ty\_2Eoption\_2Eoption$$
$$(ty\_2Elist\_2Elist\ A\_27a))^{(ty\_2Ellist\_2Ellist\ A\_27a)})^{ty\_2Enum\_2Enum})$$
$$(23)$$

**Definition 32** *We define* c_2Ellist_2EtoList *to be* $\lambda A\_27a : \iota.\lambda V0ll \in (ty\_2Ellist\_2Ellist\ A\_27a).(ap\ (ap\ (ap$

Let $c\_2Elist\_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELENGTH\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Elist\_2Elist\ A\_27a)})$$
$$(24)$$

**Definition 33** *We define* c_2Epath_2Efinite *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0sigma \in (ty\_2Epath\_2Epath\ A.$

**Definition 34** *We define* c_2Epath_2Elength *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})})$$
$$(25)$$

**Definition 35** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC$$
$$A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})})$$
$$(26)$$

**Definition 36** *We define* c_2Epath_2EPL *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

**Definition 37** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 38** *We define* c_2Epath_2EfirstP__at *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1p \in (ty\_2E$

**Definition 39** *We define* c_2Epath_2Eexists *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1p \in (ty\_2Epat$

**Definition 40** *We define* c_2Epath_2Eevery *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1p \in (ty\_2Epat$

Let $c\_2Epair\_2EFST : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epair\_2EFST$$
$$A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)})$$
$$\tag{27}$$

**Definition 41** *We define* c_2Epath_2Efirst *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A$

Let $c\_2Epath\_2EtoPath : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epath\_2EtoPath$$
$$A\_27a\ A\_27b \in ((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ (ty\_2Ellist\_2Ellist\ (ty\_2Epair\_2Eprod}$$
$$\tag{28}$$

**Definition 42** *We define* c_2Epath_2Epcons *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1r \in A\_27b.\lambda V2p$

**Definition 43** *We define* c_2Epath_2Estopped__at *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.(ap\ (c\_2Epath$

Let $c\_2Epath\_2Efilter : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}\forall A\_27b.nonempty\ A\_27b{\Rightarrow}c\_2Epath\_2Efilter$$
$$A\_27a\ A\_27b \in (((ty\_2Epath\_2Epath\ A\_27a\ A\_27b)^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)})^{(2^{A\_27a})})$$
$$\tag{29}$$

Assume the following.
$$True \tag{30}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p\ V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \tag{31}$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \tag{32}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\vee(\neg(p\ V0t)))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))) \tag{39}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{42}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ p\ V0t)))))))) \tag{43}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ A\_27a.(((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF}) \\ V0t1)\ V1t2) = V1t2)))) \tag{44}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\forall V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (45)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg(\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg(p\ (ap\ V0P\ V2x)))))) \quad (46)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \vee (\exists V4x \in A\_27a.(p\ (ap\ V1Q\ V4x)))))))) \quad (47)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.(((\exists V2x \in A\_27a.(p\ (ap\ V0P\ V2x))) \vee (p\ V1Q)) \Leftrightarrow (\exists V3x \in A\_27a.((p\ (ap\ V0P\ V3x)) \vee (p\ V1Q)))))) \quad (48)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).(((p\ V0P) \vee (\exists V2x \in A\_27a.(p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\exists V3x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V3x)))))))) \quad (49)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1Q \in 2.((\exists V2x \in A\_27a.((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A\_27a.(p\ (ap\ V0P\ V3x))) \wedge (p\ V1Q)))))) \quad (50)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\exists V2x \in A\_27a.((p\ V0P) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \wedge (\exists V3x \in A\_27a.(p\ (ap\ V1Q\ V3x)))))))) \quad (51)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ (ap\ V1P\ V2x)) \vee (p\ V0Q))) \Leftrightarrow ((\forall V3x \in A\_27a.(p\ (ap\ V1P\ V3x))) \vee (p\ V0Q))))) \quad (52)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A\_27a}).((\forall V2x \in A\_27a.((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A\_27a.(p\ (ap\ V1Q\ V3x)))))))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \lor (p\ V1B)) \lor (p\ V2C))) \Leftrightarrow (((p\ V0A) \lor (p\ V1B)) \lor (p\ V2C)))))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \lor (p\ V1B)) \Leftrightarrow ((p\ V1B) \lor (p\ V0A)))))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \lor (p\ V1B)) \Leftrightarrow ((p\ V1B) \lor (p\ V0A)))))) \tag{56}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \land (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \lor (\neg(p\ V1B)))) \land ((\neg((p\ V0A) \lor (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \land (\neg(p\ V1B))))))) \tag{57}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \lor (p\ V1B)))))) \tag{58}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \land (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{59}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \land ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27))))))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\ (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. \\ (\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \land (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \land \\ ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a) \\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27) \\ V5y\_27)))))))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in (2^{A\_27a}).(\forall V1v \in \\ A\_27a.((\forall V2x \in A\_27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ ( \\ ap\ V0f\ V1v))))) \tag{62}$$

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Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0P \in ((2^{A\_27b})^{A\_27a}).((\forall V1x \in A\_27a.(\exists V2y \in$$
$$A\_27b.(p\ (ap\ (ap\ V0P\ V1x)\ V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}).($$
$$\forall V4x \in A\_27a.(p\ (ap\ (ap\ V0P\ V4x)\ (ap\ V3f\ V4x))))))) \tag{63}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in$$
$$A\_27a.((ap\ (ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)$$
$$V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap$$
$$(ap\ (ap\ \mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}$$
$$A\_27a)\ V0x) = V0x)) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(((ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}$$
$$A\_27a\ A\_27b)\ V0x) = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V1y)) \Leftrightarrow$$
$$(V0x = V1y)))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath$$
$$A\_27a\ A\_27b).(\forall V3y \in A\_27a.(\forall V4s \in A\_27b.(\forall V5q \in$$
$$(ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(((ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}$$
$$A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p) = (ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}\ A\_27a$$
$$A\_27b)\ V3y)\ V4s)\ V5q)) \Leftrightarrow ((V0x = V3y) \wedge ((V1r = V4s) \wedge (V2p = V5q)))))))))) \tag{67}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2r \in A\_27b.(\forall V3p \in$$
$$(ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((\neg ((ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}$$
$$A\_27a\ A\_27b)\ V0x) = (ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1y)$$
$$V2r)\ V3p))) \wedge (\neg ((ap\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V1y)$$
$$V2r)\ V3p) = (ap\ (\mathsf{c\_2Epath\_2Estopped\_\_at}\ A\_27a\ A\_27b)\ V0x)))))))) \tag{68}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0P \in (2^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}).(\forall V1Q \in$$
$$(2^{A\_27a}).(((\forall V2x \in A\_27a.((p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_at}$$
$$A\_27a\ A\_27b)\ V2x))) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \wedge (\forall V3x \in A\_27a.(\forall V4r \in$$
$$A\_27b.(\forall V5p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ V0P$$
$$(ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V3x)\ V4r)\ V5p))) \Rightarrow ((p$$
$$(ap\ V1Q\ V3x)) \wedge (p\ (ap\ V0P\ V5p))))))))) \Rightarrow (\forall V6p \in (ty\_2Epath\_2Epath$$
$$A\_27a\ A\_27b).((p\ (ap\ V0P\ V6p)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Epath\_2Eevery}\ A\_27a$$
$$A\_27b)\ V1Q)\ V6p)))))))$$

(69)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0Q \in (2^{A\_27a}).(\forall V1P \in (2^{ty\_2Epath\_2Epath\ A\_27a\ A\_27b}).$$
$$((((\forall V2x \in A\_27a.((p\ (ap\ V0Q\ V2x)) \Rightarrow (p\ (ap\ V1P\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_at}$$
$$A\_27a\ A\_27b)\ V2x))))) \wedge ((\forall V3x \in A\_27a.(\forall V4r \in A\_27b.$$
$$(\forall V5p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).((p\ (ap\ V0Q\ V3x)) \Rightarrow$$
$$(p\ (ap\ V1P\ (ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V3x)\ V4r)\ V5p))))))) \wedge$$
$$(\forall V6x \in A\_27a.(\forall V7r \in A\_27b.(\forall V8p \in (ty\_2Epath\_2Epath$$
$$A\_27a\ A\_27b).((p\ (ap\ V1P\ V8p)) \Rightarrow (p\ (ap\ V1P\ (ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}$$
$$A\_27a\ A\_27b)\ V6x)\ V7r)\ V8p)))))))))) \Rightarrow (\forall V9p \in (ty\_2Epath\_2Epath$$
$$A\_27a\ A\_27b).((p\ (ap\ (ap\ \mathsf{c\_2Epath\_2Eexists}\ A\_27a\ A\_27b)\ V0Q)\ V9p)) \Rightarrow$$
$$(p\ (ap\ V1P\ V9p)))))))$$

(70)

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ($$
$$\forall V0P \in (2^{A\_27a}).((\forall V1x \in A\_27a.((p\ (ap\ V0P\ V1x)) \Rightarrow$$
$$((ap\ (ap\ (\mathsf{c\_2Epath\_2Efilter}\ A\_27a\ A\_27b)\ V0P)\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_at}$$
$$A\_27a\ A\_27b)\ V1x)) = (ap\ (\mathsf{c\_2Epath\_2Estopped\_at}\ A\_27a\ A\_27b)\ V1x)))) \wedge$$
$$(\forall V2x \in A\_27a.(\forall V3r \in A\_27b.(\forall V4p \in (ty\_2Epath\_2Epath$$
$$A\_27a\ A\_27b).((ap\ (ap\ (\mathsf{c\_2Epath\_2Efilter}\ A\_27a\ A\_27b)\ V0P)\ (ap$$
$$(ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}\ A\_27a\ A\_27b)\ V2x)\ V3r)\ V4p)) = (ap\ (ap\ ($$
$$ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Epath\_2Epath\ A\_27a\ A\_27b))\ (ap\ V0P\ V2x))$$
$$(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ (ty\_2Epath\_2Epath\ A\_27a\ A\_27b))\ ($$
$$ap\ (ap\ (\mathsf{c\_2Epath\_2Eexists}\ A\_27a\ A\_27b)\ V0P)\ V4p))\ (ap\ (ap\ (ap\ \mathsf{c\_2Epath\_2Epcons}$$
$$A\_27a\ A\_27b)\ V2x)\ V3r)\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Efilter}\ A\_27a\ A\_27b)\ V0P)$$
$$V4p)))\ (ap\ (\mathsf{c\_2Epath\_2Estopped\_at}\ A\_27a\ A\_27b)\ V2x)))\ (ap\ (ap$$
$$(\mathsf{c\_2Epath\_2Efilter}\ A\_27a\ A\_27b)\ V0P)\ V4p))))))))$$

(71)

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)))$$

(72)

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False)))$$

(73)

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Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\lor(p\ V1B)))\Rightarrow False)\Leftrightarrow \\ (((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))) \tag{74}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\lor(p\ V1B)))\Rightarrow False)\Leftrightarrow \\ ((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False)))))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( \\ (p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor((p\ V1q)\lor(p\ V2r)))\land(((p\ V0p)\lor((\neg( \\ p\ V2r))\lor(\neg(p\ V1q))))\land(((p\ V1q)\lor((\neg(p\ V2r))\lor(\neg(p\ V0p))))\land((p\ V2r)\lor \\ ((\neg(p\ V1q))\lor(\neg(p\ V0p)))))))))))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( \\ (p\ V1q)\land(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor((\neg(p\ V1q))\lor(\neg(p\ V2r))))\land(((p\ V1q)\lor \\ (\neg(p\ V0p)))\land((p\ V2r)\lor(\neg(p\ V0p))))))))))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( \\ (p\ V1q)\lor(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(\neg(p\ V1q)))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land \\ ((p\ V1q)\lor((p\ V2r)\lor(\neg(p\ V0p))))))))))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow( \\ (p\ V1q)\Rightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\lor(p\ V1q))\land(((p\ V0p)\lor(\neg(p\ V2r)))\land(( \\ \neg(p\ V1q))\lor((p\ V2r)\lor(\neg(p\ V0p))))))))))) \tag{80}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p\ V0p)\Leftrightarrow(\neg(p\ V1q)))\Leftrightarrow(((p\ V0p)\lor \\ (p\ V1q))\land((\neg(p\ V1q))\lor(\neg(p\ V0p))))))) \tag{81}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow( \\ \forall V0P \in (2^{A\_27a}).(\forall V1p \in (ty\_2Epath\_2Epath\ A\_27a \\ A\_27b).((p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eexists}\ A\_27a\ A\_27b)\ V0P)\ V1p))\Rightarrow \\ (p\ (ap\ (ap\ (\mathsf{c\_2Epath\_2Eevery}\ A\_27a\ A\_27b)\ V0P)\ (ap\ (ap\ (c\_2Epath\_2Efilter \\ A\_27a\ A\_27b)\ V0P)\ V1p))))))$$