

thm\_2Epath\_2Efinite\_drop  
(TMYPyPiE9XMP3JzXWA774Hu6XphBsy6ngG)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \tag{1}$$

Let  $ty\_2Ellist\_2Ellist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ellist\_2Ellist A0) \tag{2}$$

Let  $ty\_2Epath\_2Epath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epath\_2Epath A0 A1) \tag{3}$$

Let  $c\_2Epath\_2EfromPath : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2EfromPath A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a (ty\_2Ellist\_2Ellist (ty\_2Epair\_2Eprod A\_27b A\_27a)))^{(ty\_2Epath\_2Epath A\_27a A\_27b)}) \tag{4}$$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2ESND A\_27a A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod A\_27a A\_27b)} \tag{5}$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{6}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{7}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{8}$$

**Definition 6** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 7** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{9}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{10}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ ($

Let  $c\_2Earithmic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

**Definition 9** We define  $c\_2Earithmic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmic\_2E\_2B\ ($

**Definition 10** We define  $c\_2Earithmic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{12}$$

Let  $ty\_2Eoption\_2Eoption : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eoption\_2Eoption\ A0) \tag{13}$$

Let  $c\_2Ellist\_2Ellist\_rep : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ellist\_2Ellist\_rep\ A\_27a \in (((ty\_2Eoption\_2Eoption\ A\_27a)^{ty\_2Enum\_2Enum})^{(ty\_2Ellist\_2Ellist\ A\_27a)}) \tag{14}$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{15}$$

**Definition 11** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2.))$   
Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Esum\_2Esum A0 A1) \quad (16)$$

Let  $c\_2Esum\_2EABS\_sum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Esum\_2EABS\_sum A\_27a A\_27b \in ((ty\_2Esum\_2Esum A\_27a A\_27b)^{((2^{A\_27b})^{A\_27a})^2}) \quad (17)$$

**Definition 12** We define  $c\_2Esum\_2EINL$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27a.(ap (c\_2Esum\_2EABS\_sum A\_27a A\_27b) V0e)$

Let  $c\_2Eoption\_2Eoption\_ABS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Eoption\_2Eoption\_ABS A\_27a \in ((ty\_2Eoption\_2Eoption A\_27a)^{(ty\_2Esum\_2Esum A\_27a ty\_2Eone\_2Eone)}) \quad (18)$$

**Definition 13** We define  $c\_2Eoption\_2ESOME$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) V0x)$

**Definition 14** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 15** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$

Let  $c\_2Ellist\_2Ellist\_abs : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ellist\_2Ellist\_abs A\_27a \in ((ty\_2Ellist\_2Ellist A\_27a)^{(ty\_2Eoption\_2Eoption A\_27a)^{ty\_2Enum\_2Enum}}) \quad (19)$$

**Definition 16** We define  $c\_2Ellist\_2ELCONS$  to be  $\lambda A\_27a : \iota.\lambda V0h \in A\_27a.\lambda V1t \in (ty\_2Ellist\_2Ellist A\_27a) V0h$

**Definition 17** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A\_27a) V0P)))$

**Definition 18** We define  $c\_2Eone\_2Eone$  to be  $(ap (c\_2Emin\_2E\_40 ty\_2Eone\_2Eone) (\lambda V0x \in ty\_2Eone\_2Eone.))$

**Definition 19** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap (c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_3F) V0t))$

**Definition 20** We define  $c\_2Esum\_2EINR$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0e \in A\_27b.(ap (c\_2Esum\_2EABS\_sum A\_27a A\_27b) V0e)$

**Definition 21** We define  $c\_2Eoption\_2ENONE$  to be  $\lambda A\_27a : \iota.(ap (c\_2Eoption\_2Eoption\_ABS A\_27a) (c\_2Eone\_2Eone))$

**Definition 22** We define  $c\_2Ellist\_2ELNIL$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ellist\_2Ellist\_abs A\_27a) (\lambda V0n \in ty\_2Ellist\_2Ellist A\_27a.))$

**Definition 23** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2.))$

**Definition 24** We define  $c\_2Ellist\_2ELFINITE$  to be  $\lambda A\_27a : \iota.(\lambda V0a0 \in (ty\_2Ellist\_2Ellist A\_27a).(ap (c\_2Ellist\_2Ellist\_abs A\_27a) V0a0))$

**Definition 25** We define  $c\_2Epath\_2Efinite$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0sigma \in (ty\_2Epath\_2Epath A\_27a A\_27b)$



**Definition 33** We define  $c\_Epath\_2Elength$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0p \in (ty\_2Epath\_2Epath A\_27a A\_27b)$

**Definition 34** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a A\_27b)})^{A\_27b} \end{aligned} \quad (27)$$

**Definition 35** We define  $c\_Epath\_2EPL$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0p \in (ty\_2Epath\_2Epath A\_27a A\_27b)$

Let  $c\_2Epath\_2Etail : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2Etail \\ A\_27a A\_27b \in ((ty\_2Epath\_2Epath A\_27a A\_27b)^{(ty\_2Epath\_2Epath A\_27a A\_27b)}) \end{aligned} \quad (28)$$

Let  $c\_2Epath\_2Edrop : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epath\_2Edrop \\ A\_27a A\_27b \in (((ty\_2Epath\_2Epath A\_27a A\_27b)^{(ty\_2Epath\_2Epath A\_27a A\_27b)})^{ty\_2Enum\_2Enum}) \end{aligned} \quad (29)$$

**Definition 36** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

**Definition 37** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota. (\lambda V0x \in A\_27a. (\lambda V1f \in (2^{A\_27a}). (ap V1f V0x)))$

**Definition 38** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota. \lambda V0x \in A\_27a. \lambda V1s \in (2^{A\_27a}). (ap (c\_2Ebool\_2EIN A\_27a) V1s)$

**Definition 39** We define  $c\_2Epred\_set\_2EIMAGE$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0f \in (A\_27b^{A\_27a}). \lambda V1s \in (2^{A\_27b})$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \tag{40}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\
& (\forall V1n \in ty\_2Enum\_2Enum.((p (ap\ V0P\ V1n)) \Rightarrow (p (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\
& V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap\ V0P\ V2n))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0P \in (2^{(ty\_2Epath\_2Epath\ A\_27a\ A\_27b)}).((\forall V1p \in \\
& \quad (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(p\ (ap\ V0P\ V1p))) \Leftrightarrow ((\forall V2x \in \\
& \quad A\_27a.(p\ (ap\ V0P\ (ap\ (c\_2Epath\_2Estopped\_at\ A\_27a\ A\_27b)\ V2x)))) \wedge \\
& \quad (\forall V3x \in A\_27a.(\forall V4r \in A\_27b.(\forall V5p \in (ty\_2Epath\_2Epath \\
& \quad A\_27a\ A\_27b).(p\ (ap\ V0P\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns\ A\_27a\ A\_27b) \\
& \quad V3x)\ V4r)\ V5p))))))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (\forall V0x \in A\_27a.((p\ (ap\ (c\_2Epath\_2Efinite\ A\_27a\ A\_27b)\ (ap \\
& \quad (c\_2Epath\_2Estopped\_at\ A\_27a\ A\_27b)\ V0x))) \Leftrightarrow True)) \wedge (\forall V1x \in \\
& \quad A\_27a.(\forall V2r \in A\_27b.(\forall V3p \in (ty\_2Epath\_2Epath\ A\_27a \\
& \quad A\_27b).((p\ (ap\ (c\_2Epath\_2Efinite\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\
& \quad A\_27a\ A\_27b)\ V1x)\ V2r)\ V3p)))) \Leftrightarrow (p\ (ap\ (c\_2Epath\_2Efinite\ A\_27a\ A\_27b) \\
& \quad V3p))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0x \in A\_27a.(\forall V1r \in A\_27b.(\forall V2p \in (ty\_2Epath\_2Epath \\
& \quad A\_27a\ A\_27b).((ap\ (c\_2Epath\_2Etail\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\
& \quad A\_27a\ A\_27b)\ V0x)\ V1r)\ V2p))) = V2p)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad (\forall V0x \in A\_27a.((ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ (ap\ (c\_2Epath\_2Estopped\_at \\
& \quad A\_27a\ A\_27b)\ V0x))) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ ty\_2Enum\_2Enum) \\
& \quad c\_2Enum\_2E0)\ (c\_2Epred\_set\_2EEMPTY\ ty\_2Enum\_2Enum)))) \wedge (\forall V1x \in \\
& \quad A\_27a.(\forall V2r \in A\_27b.(\forall V3q \in (ty\_2Epath\_2Epath\ A\_27a \\
& \quad A\_27b).((ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ (ap\ (ap\ (ap\ (c\_2Epath\_2Epcns \\
& \quad A\_27a\ A\_27b)\ V1x)\ V2r)\ V3q))) = (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ ty\_2Enum\_2Enum) \\
& \quad c\_2Enum\_2E0)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum) \\
& \quad c\_2Enum\_2ESUC)\ (ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b)\ V3q))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0p \in (ty\_2Epath\_2Epath\ A\_27a\ A\_27b).(p\ (ap\ (ap\ (c\_2Ebool\_2EIN \\
& \quad ty\_2Enum\_2Enum)\ c\_2Enum\_2E0)\ (ap\ (c\_2Epath\_2EPL\ A\_27a\ A\_27b) \\
& \quad V0p))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & (\forall V0p \in (ty\_2Epath\_2Epath\ A.27a\ A.27b).((ap\ (ap\ (c\_2Epath\_2Edrop \\ & A.27a\ A.27b)\ c\_2Enum\_2E0)\ V0p) = V0p)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\ & (\forall V2p \in (ty\_2Epath\_2Epath\ A.27a\ A.27b).((ap\ (ap\ (c\_2Epath\_2Edrop \\ & A.27a\ A.27b)\ (ap\ c\_2Enum\_2ESUC\ V1n))\ V2p) = (ap\ (ap\ (c\_2Epath\_2Edrop \\ & A.27a\ A.27b)\ V1n)\ (ap\ (c\_2Epath\_2Etail\ A.27a\ A.27b)\ V2p)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\neg(p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V0x)\ (c\_2Epred\_set\_2EEMPTY\ A.27a)))))) \quad (49)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.(\forall V2s \in (2^{A.27a}).((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V0x)\ (ap\ (ap\ (c\_2Epred\_set\_2EINSERT\ A.27a)\ V1y)\ V2s))) \Leftrightarrow ((V0x = \\ & V1y) \vee (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V0x)\ V2s)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0y \in A.27b.(\forall V1s \in (2^{A.27a}).(\forall V2f \in (A.27b^{A.27a}). \\ & ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27b)\ V0y)\ (ap\ (ap\ (c\_2Epred\_set\_2EIMAGE \\ & A.27a\ A.27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A.27a.((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A.27a)\ V3x)\ V1s)))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ c\_2Enum\_2ESUC\ V1n)) \Leftrightarrow (V0m = V1n)))) \quad (52)$$

**Theorem 1**

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow ( \\ & \forall V0p \in (ty\_2Epath\_2Epath\ A.27a\ A.27b).(\forall V1n \in ty\_2Enum\_2Enum. \\ & ((p\ (ap\ (ap\ (c\_2Ebool\_2EIN\ ty\_2Enum\_2Enum)\ V1n)\ (ap\ (c\_2Epath\_2EPL \\ & A.27a\ A.27b)\ V0p))) \Rightarrow ((p\ (ap\ (c\_2Epath\_2Efinite\ A.27a\ A.27b)\ (ap \\ & (ap\ (c\_2Epath\_2Edrop\ A.27a\ A.27b)\ V1n)\ V0p))) \Leftrightarrow (p\ (ap\ (c\_2Epath\_2Efinite \\ & A.27a\ A.27b)\ V0p)))))) \end{aligned}$$